Math 253: Quiz 4 **16 February, 2023**

Name: _____ **/ 25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[4 points]** Find the arc length of the vector-valued function $\mathbf{r}(t) = -t\,\mathbf{i} + 4t\,\mathbf{j} + 3t\,\mathbf{k}$ over $[0,1]$. (*Hint.* You can do this either way, with or without an integral.)

2. **[4 points]** Compute the arc-length function $s(t)$ for the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ from $t = 0$.

3. [5 points] Explain in 2 or 3 complete sentences what the following definition of curvature, given in section 3.3, is saying:

$$\kappa(s) = \left\| \frac{d\mathbf{T}}{ds} \right\|.$$

(*Hint.* What are the objects on the right side? Use the phrase "rate of change" where appropriate. And what is the curvature geometrically?)

4. [4 points] Find the level surface of the three-variable function $w(x,y,z) = x^2 + y^2 + z^2$ at $c = 36$. Describe this surface in a complete sentence.

5. [4 points] Find and sketch the domain of the function $f(x,y) = \sqrt{4-x^2-y^2}$.

6. [4 points] Visualize the same function $f(x,y) = \sqrt{4-x^2-y^2}$ by finding and sketching at least three level curves. Label the curves with their function value, that is, their "c" value.

Extra Credit. [1 point] Given the definition $\kappa(s) = \left\| \dfrac{d\mathbf{T}}{ds} \right\|$, show that $\kappa(t) = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$ for a vector-valued function $\mathbf{r}(t)$.

EXTRA SPACE FOR ANSWERS