Math 253: Quiz 3 **9 February, 2023**

Name: ______________________________ | / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [5 points] Suppose $\mathbf{r}(t) = \langle t, \cos t, \sin t \rangle$.

a) Sketch this three-dimensional (space) curve.

b) Is $\mathbf{r}(t)$ continuous at $t = \pi$? If so, explain why in a few words, and state as an equation.

2. [4 points] Consider the vector-valued function $\mathbf{r}(t) = t^2\,\mathbf{i} + \sqrt{t-3}\,\mathbf{j} + \frac{5}{2t-1}\,\mathbf{k}$.

a) What is the domain of $\mathbf{r}(t)$?

b) Compute $\lim_{t \to 3^+} \mathbf{r}(t)$.

3. [8 points] Suppose that a moving particle has position function $\mathbf{r}(t) = \langle e^{-t}, t, te^{-t} \rangle$.

a) What is the velocity $\mathbf{v}(t)$ at time $t = 1$?

b) Calculate the tangent line to the curve $\mathbf{r}(t)$ at $t = 1$.

c) Compute $\int_0^1 \mathbf{r}(t)\,dt =$

4. [8 points] The vector-valued function $\mathbf{r}(t) = a\sin(\omega t)\,\mathbf{i} + a\cos(\omega t)\,\mathbf{j}$, for constants $a > 0$ and $\omega > 0$, describes uniform circular motion.

a) Show that the velocity $\mathbf{v}(t)$ at any time is orthogonal to the position $\mathbf{r}(t)$.

b) Show that the acceleration $\mathbf{a}(t)$ at any time is orthogonal to the velocity $\mathbf{v}(t)$.

Extra Credit. [1 point] Show that

$$\frac{d}{dt}\left(\frac{1}{2}\|\mathbf{r}(t)\|^2\right) = \mathbf{r}(t)\cdot\mathbf{r}'(t)$$

EXTRA SPACE FOR ANSWERS