Name: SOLUTIONS

/ 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [4 points] Compute $\mathbf{u} \times \mathbf{v}$ if $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = \mathbf{j} + 2\mathbf{k}$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{vmatrix} = (3\cdot 2 - 0)\hat{\imath} - (2\cdot 2 - 0)\hat{\jmath} + (2\cdot 1 - 0)\hat{k}$$

$$= 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k} = \langle 6, -4, 2 \rangle$$

either form is just fine!

2. [4 points] Suppose $P(0,0,0)$ is a point in the plane with normal vector $\mathbf{n} = \langle -3, 2, -1 \rangle$. Find the general equation of the plane. Express your answer in the form $ax + by + cz + d = 0$.

if $Q(x,y,z)$ is in the plane then

$$\vec{n} \cdot \overrightarrow{PQ} = 0$$

$$\Leftrightarrow \langle -3, 2, -1 \rangle \cdot \langle x-0, y-0, z-0 \rangle = 0$$

$$\Leftrightarrow -3x + 2y - z = 0$$

$a = -3$, $b = 2$, $c = -1$, $d = 0$

3. [4 points] Find a general equation of the plane through the three points $P(3,-1,2)$, $Q(1,0,1)$, and $R(0,-1,1)$. Express your answer in the form $ax+by+cz+d=0$.

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 1 & -1 \\ -3 & 0 & -1 \end{vmatrix} = (-1-0)\hat{\imath} - (2-3)\hat{\jmath} + (0+3)\hat{k}$$

$$= \langle -1, 1, 3 \rangle$$

$S(x,y,z)$ is point in plane

$$\vec{n} \cdot \vec{PS} = 0$$

$$\iff \langle -1, 1, 3 \rangle \cdot \langle x-3, y+1, z-2 \rangle = 0$$

$$\iff -(x-3) + y+1 + 3(z-2) = 0$$

$$\iff -x + y + 3z - 2 = 0$$

check P, Q, R satisfy this? ✓

4. [5 points] Consider the line passing through the two points $P(4,0,5)$ and $Q(2,3,1)$.

a) Find a vector equation of the line.

$$\vec{r}(t) = \vec{p} + t\,\vec{PQ} = \langle 4,0,5 \rangle + t\langle -2,3,-4 \rangle$$

$$= \underbrace{\langle 4-2t, 0+3t, 5-4t \rangle}_{\text{also fine}}$$

b) Find parametric equations of the line.

$$x(t) = 4-2t$$
$$y(t) = 3t$$
$$z(t) = 5-4t$$

5. [4 points] Consider points $A(3,-1,2)$, $B(1,0,1)$, and $C(0,-1,1)$. Find the area of the triangle ABC.

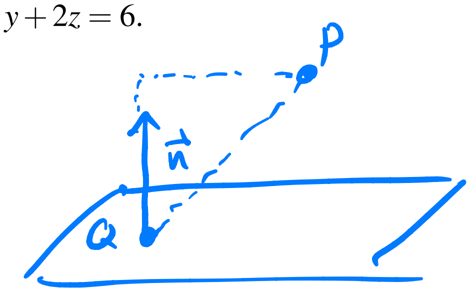
$$A_\Delta = \frac{1}{2} \| \vec{AB} \times \vec{AC} \|$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -1 \\ -3 & 0 & -1 \end{vmatrix} = (-1-0)\hat{i} - (2-3)\hat{j} + (0+3)\hat{k} = \langle -1, 1, 3 \rangle$$

so:

$$A_\Delta = \frac{1}{2}\sqrt{1^2+1^2+3^2} = \frac{1}{2}\sqrt{11} = \frac{\sqrt{11}}{2}$$

6. [4 points] Find the distance from the point $P(1,5,-4)$ to the plane $3x-y+2z=6$.



$\vec{n} = \langle 3, -1, 2 \rangle$ for the plane

$Q = (2, 0, 0)$ is in the plane

$$d = |\text{comp}_{\vec{n}} \vec{QP}| = \left| \frac{\vec{QP} \cdot \vec{n}}{\|\vec{n}\|} \right|$$

$$= \left| \frac{\langle -1, 5, -4 \rangle \cdot \langle 3, -1, 2 \rangle}{\sqrt{3^2+1^2+2^2}} \right| = \left| \frac{-3-5-8}{\sqrt{14}} \right|$$

$$= \frac{16}{\sqrt{14}} = \frac{8\sqrt{2}}{\sqrt{7}}$$

Extra Credit. [1 point] Show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to $\mathbf{u} + \mathbf{v}$ and also to $\mathbf{u} - \mathbf{v}$.

remember that $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$ and $\vec{v}$,

so:

$$(\vec{u} \times \vec{v}) \cdot (\vec{u} + \vec{v}) = \overset{0}{\cancel{(\vec{u} \times \vec{v}) \cdot \vec{u}}} + \overset{0}{\cancel{(\vec{u} \times \vec{v}) \cdot \vec{v}}} = 0$$

$$(\vec{u} \times \vec{v}) \cdot (\vec{u} - \vec{v}) = \overset{0}{\cancel{(\vec{u} \times \vec{v}) \cdot \vec{u}}} - \overset{0}{\cancel{(\vec{u} \times \vec{v}) \cdot \vec{v}}} = 0.$$

EXTRA SPACE FOR ANSWERS