Math 253: Quiz 10 **Thursday 20 April, 2023**

Name: ____________________ **/ 25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [4 points] Is $\mathbf{F} = 2x\cos y\,\mathbf{i} - x^2 \sin y\,\mathbf{j}$ conservative? If it is, find a potential function.

2. [4 points] Is $\mathbf{F} = \langle \sin y, -x\cos y, x \rangle$ conservative? If it is, find a potential function.

3. **[4 points]** Is the following statement **true** or **false**? Explain in one or two sentences, justifying with any theorem that might apply.

> If a vector field $\mathbf{F}(x,y,z)$ is conservative on the open and connected region D, then line integrals of $\mathbf{F}$ are path independent on D, regardless of the shape of D.

4. **[4 points]** Suppose $f(x,y,z) = xyz^2 - yz$ and C is a straight line from $(0,1,2)$ to $(1,1,1)$. Evaluate the integral using the Fundamental Theorem for Line Integrals:

$$\int_C \nabla f \cdot d\mathbf{r} =$$

5. [4 points] Suppose C is the boundary of the region lying between the graphs of $y = 0$ and $y = 1 - x^2$, and assume that C is oriented in the counterclockwise direction. Compute using Green's theorem:

$$\oint_C x^2\,dx + 3yx\,dy =$$

6. [5 points] Suppose D is **any** simply-connected region in the plane, and let C be its boundary, oriented in the counterclockwise direction. Compute using Green's theorem and simplify as far as possible:

$$\oint_C -y\,dx + x\,dy =$$

Extra Credit. [1 point] Suppose C is the parameterized curve $\mathbf{r}(t) = \left\langle \cos(1-t^2), \sin(1-t^2) \right\rangle$ for $-1 \le t \le 1$, whose graph is the upper half of the unit circle. Suppose $\mathbf{F} = \langle xe^y, \sin(x+y) \rangle$. Compute the line integral, explaining your steps:

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

(**Hint.** Do not apply brute force. Think about the curve.)

EXTRA SPACE FOR ANSWERS