Name: _____ | **/ 25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[8 points]** Suppose we have three vectors, $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{j} + 3\mathbf{k}$, $\mathbf{c} = -\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$. Compute the following quantities which are either scalars or vectors. You can write the vectors using either component notation or standard unit vector notation.

   a) $\|\mathbf{a}\| =$

   b) $(\mathbf{a} \cdot \mathbf{b})\,\mathbf{c} =$

   c)   a unit vector in the direction of $\mathbf{b}$:

       $\mathbf{u} =$

   *d)*   the vector projection of $\mathbf{b}$ onto $\mathbf{a}$:

       $\mathbf{w} = \text{proj}_{\mathbf{a}}\, \mathbf{b} =$

2. [6 points] Find the equation of the sphere which has diameter PQ, where $P = (2, -1, -3)$ and $Q = (-2, 5, -1)$.

3. [6 points] Describe the set of points in three dimensional space that satisfies $x^2 + (z-2)^2 = 1$, and sketch a graph of the surface. (*Please make your graph at least two inches tall, label the axes, and put at least one scale value, a labeled tick mark, along each axis.*)

4. [5 points] A methane molecule (figure) has a carbon atom situated at the origin and four hydrogen atoms located at points $P(1, 1, -1)$, $Q(1, -1, 1)$, $R(-1, 1, 1)$, and $S(-1, -1, -1)$. Find the angle θ between vectors $\overrightarrow{OS}$ and $\overrightarrow{OR}$.

Hint. It is just fine if your answer has an $\arccos()$ *in it, but otherwise it should be simplified. I know you do not have a calculator!*
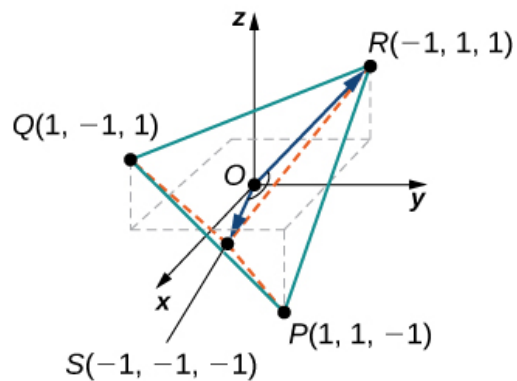
Extra Credit. [1 point] Show that $\|\mathbf{v} - \mathbf{u}\|^2 = \|\mathbf{v}\|^2 - 2\mathbf{u} \cdot \mathbf{v} + \|\mathbf{u}\|^2$.

EXTRA SPACE FOR ANSWERS