Name: ____________________________________

Math 253 Calculus III (Bueler) Thursday, 6 April 2023

Midterm Exam 2

No book, notes, electronics, calculator, or internet access. 100 points possible. 70 minutes maximum.

1. (*10 pts*) Find an equation of the tangent plane of the surface $z = \ln(10x^2 + 2y^2 + 1)$ at the point $P(0, 0, 0)$.

2. **(a)** (*5 pts*) Suppose $f(u, v)$ is a function of two variables, and that, in turn, $u = u(r, s)$ and $v = v(r, s)$. Write down the chain rule which computes $\frac{\partial f}{\partial s}$:

$$\frac{\partial f}{\partial s} =$$

(b) (*5 pts*) Specifically suppose $f(u, v) = uv$, $u(r, s) = r\cos s$, and $v(r, s) = r\sin s$. Compute the following partial derivative, and express your answer as a simplified expression in variables r, s.

$$\frac{\partial f}{\partial s} =$$

3. Consider the function $f(x, y) = x^3 + y^3 - 12x - 15y + 7$.

(a) (*5 pts*) Compute the gradient:

$\nabla f(x, y) =$

(b) (*5 pts*) Find all of the critical points. Write each one as a pair (x, y).

(c) (*5 pts*) Use the second derivative test to classify all of the critical points, as local maximum, local minimum, or saddle point.

(d) (*3 pts*) Consider the square $D = [-10, 10] \times [-10, 10]$. Does the absolute maximum of $f(x, y)$ over D occur at one of the critical points found in part **(b)**? State the answer **yes** or **no**, and explain in one sentence.

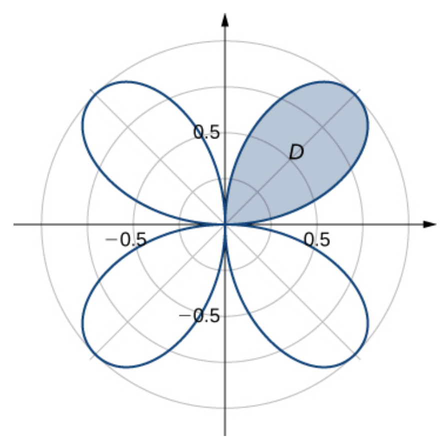
(*Hint.* You do not need to find the absolute maximum! But consider values of $f(x, y)$ when answering.)

4. **(a)** (*5 pts*) Sketch the region
$D = \Big\{(x, y)\,\Big|\, -1 \leq y \leq 0,\ 0 \leq x \leq 1 - y^2\Big\}.$

(b) (*10 pts*) Compute and simplify the double integral over the region D in part **(a)**:

$$\iint_D xy\, dA =$$

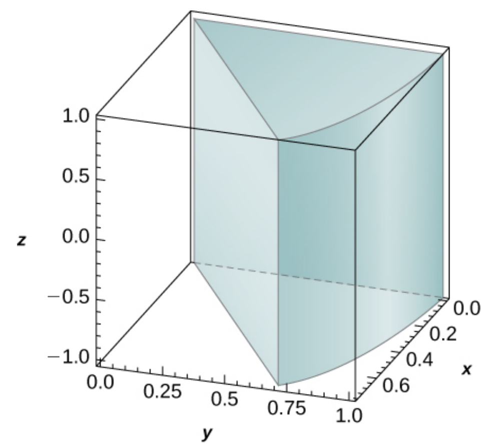
5. (*10 pts*) Set up, and then evaluate, a double integral for the area of one leaf of the rose $r = \sin(2\theta)$, as shown in the figure.



6. (*10 pts*) Define $B = \left\{(x,y,z) \,\middle|\, 0 \le x \le 1,\ 0 \le y \le \pi,\ -1 \le z \le 1\right\}$, a rectangular solid box. Evaluate the triple integral:

$$\iiint_B x \sin y \, dV =$$

7. (*10 pts*) The region $E = \left\{(x,y,z) \,\middle|\, x^2 + y^2 \le 1,\ x \ge 0,\ x \ge y,\ -1 \le z \le 1\right\}$ is shown below. Use **cylindrical coordinates** to evaluate the triple integral $\iiint_E f(x,y,z)\, dV$ if $f(x,y,z) = xy$.



8. (*10 pts*) Find the average value of the function $f(x, y, z) = x^2 + y^2 + z^2$ over the unit sphere $S = \Big\{(x, y, z) \,\Big|\, x^2 + y^2 + z^2 \leq 1\Big\}$.

(*Hints.* What coordinates would make this easiest? Yes, you *may* use the fact that the volume of the unit sphere is $4\pi/3$; there is no need to justify it.)

9. (*7 pts*) Use **spherical coordinates** to fully set up a triple integral for the volume which is outside the cone $z^2 = x^2 + y^2$ but inside the unit sphere $x^2 + y^2 + z^2 = 1$. Do not evaluate the integral.

(*Hint.* Start by drawing a decent sketch!)

Extra Credit. (*3 pts*) The equation $x^2 + y^2 = 9$ is a cylinder. Convert this equation to **spherical coordinates**, and write your simplified answer in the form $\rho = f(\varphi, \theta)$.

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