Math 252: Quiz 9 **31 March, 2022**

Name: ____________________________________ ________ / **25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[9 points]** Do the series converge absolutely, conditionally, or neither (diverge)? Show your work and circle one answer.

a. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n-2}{\sqrt{n}}$

CONVERGES ABSOLUTELY   CONVERGES CONDITIONALLY   DIVERGES

b. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$

CONVERGES ABSOLUTELY   CONVERGES CONDITIONALLY   DIVERGES

c. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(\pi n)}{\sqrt{n}}$

CONVERGES ABSOLUTELY   CONVERGES CONDITIONALLY   DIVERGES

2. [6 points] Use the ratio or root test to determine whether the series converges or diverges. Show your work.

a. $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{3^k}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+2)^2}{n!}$

3. [3 points] How close is the partial sum $S_{10} = \displaystyle\sum_{n=1}^{10} \frac{(-1)^n}{2n+1}$ to the convergent infinite sum (series) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$? That is, how large is the remainder R_{10}? Give a brief explanation, and then answer quantitatively in the box, using the fact that the series is alternating.

$|R_{10}| \leq$ ☐

4. [4 points] Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x)^n}{n}$. Show your work.

$R =$ ☐ interval: ☐

5. [3 points] Use the geometric series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ to find a power series for the function $f(x) = \frac{x^2}{1+x^2}$.

Extra Credit. [2 points] Explain why $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$ converges conditionally.

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