Math 252: Quiz 8 **24 March, 2022**

Name: SOLUTIONS ______ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [6 points] Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

a. Use the integral test to show this series converges.

$$\int_1^\infty \frac{dx}{x^2+1} = \lim_{t\to\infty} \arctan x\Big]_1^t = \left(\lim_{t\to\infty} \arctan t\right) - \arctan 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} < \infty$$ so converges

by integral test

b. Use a comparison test to show this series converges. (*Make sure to say which series you are comparing to, and why it converges.*)

$\frac{1}{n^2+1} \le \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series)

so $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges by comparision test

2. [3 points] Consider the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p < 0$. Show that such p-series diverge by the divergence test.

$$\lim_{n\to\infty} \frac{1}{n^p} = \lim_{n\to\infty} n^{-p} = +\infty,$$

since $-p$ is a positive power, so

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges by divergence test

3. [9 points] Use the comparison test or the limit comparison test to determine whether the following series converge or diverge. (*Say which comparison test you are using, and what series you are comparing to!*)

a. $\sum_{n=1}^{\infty} \frac{2^n}{4^n - 3^n}$

$\boxed{\text{Compare: } \sum_{n=1}^{\infty} \frac{1}{2^n}}$

geometric

$$\lim_{n\to\infty} \frac{\frac{2^n}{4^n-3^n}}{\frac{1}{2^n}} = \lim_{n\to\infty} \frac{4^n}{4^n-3^n} = \lim_{n\to\infty} \frac{1}{1-(3/4)^n} = \frac{1}{1-0} = 1 = L \quad (\neq 0, \neq \infty)$$

so original series converges by limit comparison test

b. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

$\frac{\ln n}{n} \geq \frac{1}{n}$ and $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges (harmonic series) so $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ diverges by comparison test

c. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$

$|\cos n| \leq 1$ so $\frac{\cos^2 n}{n^2} \leq \frac{1}{n^2}$

and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series), so

$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$ converges by comparison test

4. [4 points] Simplify the following series, and then determine if the series converges or diverges by applying a test. (*Justify your conclusion about converge or diverge.*)

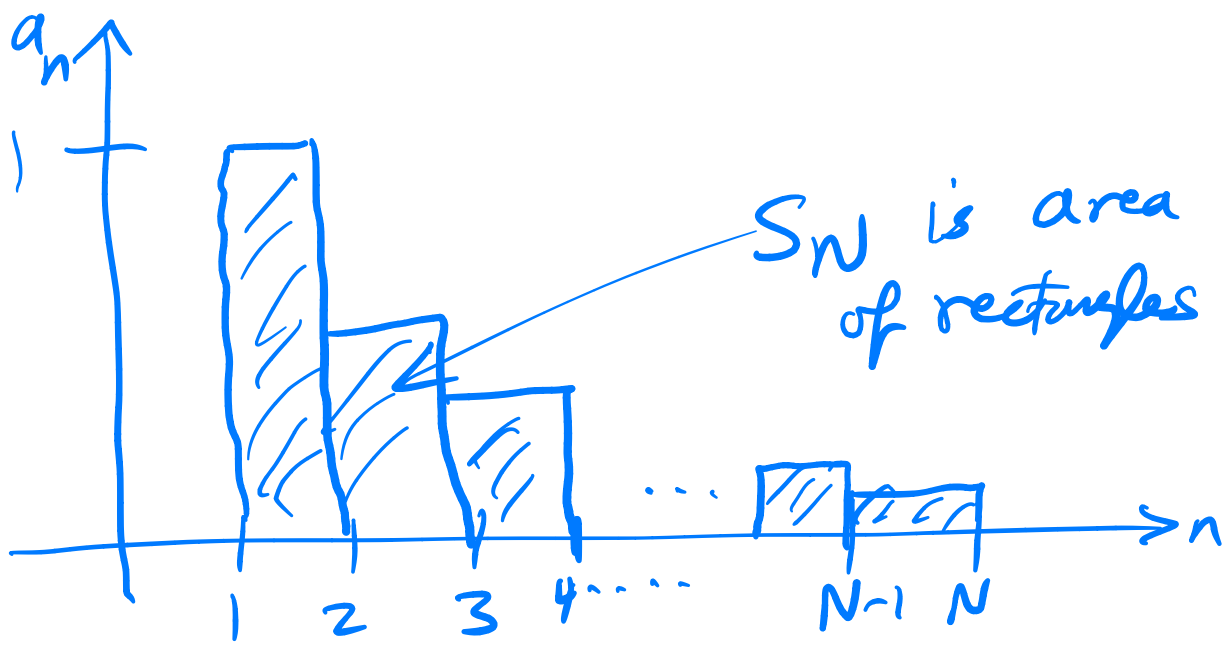
a. $\sum_{n=2}^{\infty} \frac{(n-1)!}{n!}$ $= \sum_{n=2}^{\infty} \frac{\cancel{(n-1)}\cancel{(n-2)}\cdots \cancel{2\cdot 1}}{n\cancel{(n-1)}\cancel{(n-2)}\cdots \cancel{2\cdot 1}} = \sum_{n=2}^{\infty} \frac{1}{n}$

diverge (it is p-series with $p=1$)

b. $\sum_{n=2}^{\infty} \frac{n!}{(n-1)!}$ $= \sum_{n=2}^{\infty} \frac{n(n-1)\cdots 2\cdot 1}{(n-1)\cdots 2\cdot 1} = \sum_{n=2}^{\infty} n$

diverge by divergence test $\left(\lim_{n\to\infty} n = +\infty \neq 0\right)$

5. [3 points] Sketch a partial sum S_N of the harmonic series as the total area of rectangles of width one. (*Please label axes appropriately.*)



Extra Credit. [2 points] Which of the following two series do we **know** converges or diverges? (Which is it, and why?) Explain in words why the answer is unknown for the other series.

$$\sum_{n=1}^{\infty} \frac{(\cos n)^2}{n}, \qquad \sum_{n=1}^{\infty} \frac{1}{n(\cos n)^2}$$

$\sum_{n=1}^{\infty} \frac{1}{n(\cos n)^2}$ diverges: here $(\cos n)^2 \leq 1$

so $\frac{1}{n(\cos n)^2} \geq \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

(harmonic)

to us!

$\sum_{n=1}^{\infty} \frac{(\cos n)^2}{n}$ unknown: $(\cos n)^2 \leq 1$

so $\frac{(\cos n)^2}{n} \leq \frac{1}{n}$, but $\sum \frac{1}{n}$ diverges. are terms in $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n}$ small enough to converge?

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