Name: ______________________________

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30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. **[5 points]** Suppose $a_n = \frac{1}{n}$.

   a. Find the limit of the sequence a_n.

   b. Compute and simplify the first four partial sums $S_1, \ldots, S_4$.

2. **[3 points]** Find a function $f(n)$ for the nth term a_n of the following recursively defined sequence:

$$a_1 = 3 \text{ and } a_{n+1} = 2a_n \text{ for } n \geq 1.$$

3. [3 points] Either show that the sequence diverges or, if it converges, find its limit: $a_n = \dfrac{\ln(2n)}{\ln(n^2)}$

4. [6 points] State whether the given series converges or diverges, and explain why. If the series converges, find its sum. (**Hint.** Geometric series.)

a. $1+e+e^2+e^3+\dots$

b. $1-\dfrac{1}{10}+\dfrac{1}{100}-\dfrac{1}{1000}+\dfrac{1}{10000}-\dots$

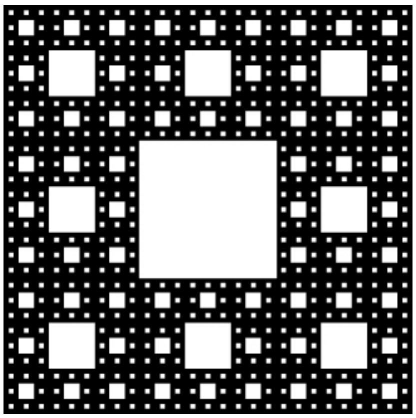
5. **[8 points]** Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}.$$

a. Use partial fractions and "telescoping" to write a simplified formula for the partial sum $S_k = \sum_{n=1}^{k} \frac{1}{n(n+1)}$.

b. Compute the value of the series.

Extra Credit. [2 points] The black thing below, called the **Sierpinksi gasket**, is built by removing all of the white part from an original fully-black square. Assume the original square has side-length one and thus area one. Remove the middle 1/9th of the area. The remainder is 8 smaller black squares. For each of these, remove the middle 1/9th. Continuing in this way, by infinitely-many stages you remove all the white area. Using geometric series, compute the white area you removed. What area is left, the black area?



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