CORRECTED 2b

Math 252: Quiz 1 SOLNS **13 January, 2022**

Name: SOLNS

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30 minutes maximum. 24 points possible; each part is worth 2 points. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{e^x}{x^3}$

$$f'(x) = \frac{e^x x^3 - e^x 3x^2}{x^6}$$

$$= \frac{e^x}{x^4}(x-3)$$

b. $f(x) = \left(\ln(x^2+e^2)\right)^5$

$$f'(x) = 5\left(\ln(x^2+e^2)\right)^4 \frac{1}{x^2+e^2} 2x$$

$$= \frac{10x\left(\ln(x^2+e^2)\right)^4}{x^2+e^2}$$

c. $f(x) = a^{\sin(x)}$ where a is a constant, $a > 1$

$$f'(x) = \ln(a)\, a^{\sin(x)} \cos(x)$$

d. $f(x) = \sec\left(\dfrac{x}{x+1}\right)$

$$f'(x) = \sec\left(\frac{x}{x+1}\right)\tan\left(\frac{x}{x+1}\right)\frac{x+1-x}{(x+1)^2}$$

$$= \boxed{\frac{\sec\left(\frac{x}{x+1}\right)\tan\left(\frac{x}{x+1}\right)}{(x+1)^2}}$$

e. $f(x) = e^{\pi x+1} + \sqrt{3}\tan(\pi x)$

$$f'(x) = \pi e^{\pi x+1} + \sqrt{3}\sec^2(\pi x)\,\pi$$

$$= \boxed{\pi\left(e^{\pi x+1} + \sqrt{3}\sec^2(\pi x)\right)}$$

f. Find $\dfrac{dy}{dx}$ if $2x + y = \cos(xy)$. (You must solve for $\dfrac{dy}{dx}$.)

$$2 + \frac{dy}{dx} = -\sin(xy)\left(1\cdot y + x\cdot\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left(1 + x\sin(xy)\right) = -y\sin(xy) - 2$$

$$\boxed{\frac{dy}{dx} = -\frac{y\sin(xy)+2}{1+x\sin(xy)}}$$

2. [12 points] Compute the following definite integrals and antiderivatives (indefinite integrals). Remember that antiderivatives need a "$+C$".

a. $\int_1^2 \frac{2+x^3}{x^2}\,dx$ $= \int_1^2 2x^{-2}+x\,dx$

$= \left[2x^{-1}+\frac{x^2}{2}\right]_1^2 = \left(-\frac{2}{2}+2\right)-\left(-2+\frac{1}{2}\right)$

$= -1+2+2-\frac{1}{2} = \frac{5}{2}$

b. $\int \frac{e^{3x}}{\sqrt{5+e^{3x}}}\,dx$

$\leftarrow \begin{cases} u = 5+e^{3x} \\ du = 3e^{3x}dx \\ \frac{du}{3} = e^{3x}dx \end{cases}$

$= \int \frac{du/3}{\sqrt{u}}$

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$= \frac{1}{3}\int u^{-1/2}du = \frac{1}{3}2u^{1/2} = \frac{2}{3}(5+e^{3x})^{1/2}+C$

c. $\int \frac{1}{x} + \sec(x)\tan(x)\,dx$

$= \ln|x| + \sec(x) + C$

d. $\int x\sqrt{2-x}\,dx$

$$\begin{cases} u = 2-x \Leftrightarrow x = 2-u \\ du = -dx \\ -du = dx \end{cases}$$

$$= -\int (2-u)\sqrt{u}\,du$$

$$= \int u^{3/2} - 2u^{1/2}\,du = \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$$

$$= \frac{2}{5}(2-x)^{5/2} - \frac{4}{3}(2-x)^{3/2} + C$$

e. $\int_0^2 e^x \cos(1+e^x)\,dx$

$$\begin{cases} u = 1+e^x \\ du = e^x\,dx \end{cases}$$

$$= \int_2^{1+e^2} \cos(u)\,du$$

$$= \sin(u)\Big]_2^{1+e^2} = \sin(1+e^2) - \sin(2)$$

f. $\int \tan(x)\sec^2(x)\,dx$

$$\begin{cases} u = \tan(x) \\ du = \sec^2(x)\,dx \end{cases}$$

$$= \int u\,du$$

$$= \frac{u^2}{2} + C = \frac{1}{2}(\tan(x))^2 + C$$