Math 252: Quiz 10 **14 April, 2022**

Name: ______________________________ ________ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [8 points] Using any convenient method, write the Maclaurin series of the given function. Use sigma notation for your answer.

a. $g(x) = xe^{-2x}$

b. $f(x) = \dfrac{\sin x}{x}$

2. [4 points] Use the answer from problem **1 b** on the previous page to compute the integral:

$$\int_0^x \frac{\sin t}{t}\,dt =$$

3. [4 points] Use the ratio (or root) test, plus a check on series convergence at the endpoints, to show that the interval of convergence of the following familiar Maclaurin series is $[-1,1]$.

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

4. [5 points] Use the binomial series for $(1+x)^r$ to write out the first four nonzero terms of

$$f(x) = (1+x^2)^{1/3}$$

(**Hint.** This means you will write the Taylor polynomial of degree 6 for $f(x)$.)

5. [4 points] Consider the parametric curve $x = 1 + \cos t$, $y = 3 - \sin t$.

a. Eliminate the parameter to convert into rectangular form.

b. Sketch the curve in the x, y plane.

Extra Credit. [2 points] Finding the antiderivative

$$\int \sqrt{x}\, e^x \, dx$$

is traditionally regarded as impossible. Indeed it is impossible if you want a finite expression in terms of familiar functions, but fairly easy if you accept a series, one which is not quite a standard power series, for the answer. Starting from the familiar Maclaurin series for e^x, find a series form of this antiderivative.

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