Math 252: Quiz 9 SOLUTIONS **10 November, 2022**

Name: ______________________________ **/ 24**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 24 points possible.

1. **[8 points]** Do the series converge absolutely, converge conditionally, or diverge? Show your work, identify tests you used, and circle one answer.

a. $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ $b_n = \frac{1}{n}$ $\lim_{n\to\infty} \frac{1}{n} = 0$, b_n dec.

so converges by AST

$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ diverge (harmonic)

CONVERGES ABSOLUTELY | CONVERGES CONDITIONALLY | DIVERGES

b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ $\left(b_n = \frac{1}{n!}\right)$ $\sum_{n=1}^{\infty} \frac{1}{n!} = \sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n!} \right|$

ratio test: $\rho = \lim_{n\to\infty} \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \lim_{n\to\infty} \frac{\cancel{n!}}{(n+1)\cancel{n!}} = \lim_{n\to\infty} \frac{1}{n+1} = 0$

Since $\rho < 0$,

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2. [8 points] Use the ratio or root test to determine whether the series converges or diverges. Show your work.

a. $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{2^k}$

root test:

$$\rho = \lim_{k\to\infty} \sqrt[k]{\frac{k^2}{2^k}} = \lim_{k\to\infty} \frac{\left(\sqrt[k]{k}\right)^2}{2}$$

$$= \frac{1^2}{2} = \frac{1}{2} < 1$$

$\therefore$ converges

[also ratio test:

$$\rho = \lim_{k\to\infty} \frac{\frac{(k+1)^2}{2^{k+1}}}{\frac{k^2}{2^k}}$$

$$= \lim_{k\to\infty} \frac{(k+1)^2 \cancel{2^k}}{\cancel{2^k}\cdot 2 \cdot k^2}$$

$$= \frac{1}{2} \lim_{k\to\infty} \frac{(k+1)^2}{k^2} = \frac{1}{2} < 1 \ldots]$$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+2)^2}{n!}$

ratio test

$$\rho = \lim_{n\to\infty} \frac{\frac{(n+3)^2}{(n+1)!}}{\frac{(n+2)^2}{n!}} = \lim_{n\to\infty} \frac{(n+3)^2\, \cancel{n!}}{(n+1)\,\cancel{n!}\,(n+2)^2}$$

$$= \lim_{n\to\infty} \frac{n^2+\ldots}{n^3+\ldots} \underset{\times 2}{\overset{LH}{=}} \lim_{n\to\infty} \frac{2}{6n+c} = 0 < 1$$

so converges

3. [8 points] Use any test to determine whether the series converges or diverges. Show your work.

a. $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\sqrt{n+1}-\sqrt{n}\right)$

$$b_n = \frac{\sqrt{n+1}-\sqrt{n}}{1} = \frac{\sqrt{n+1}-\sqrt{n}}{1}\,\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$$

$$= \frac{(n+1)-n}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

so $\lim_{n\to\infty} b_n = \frac{1}{\infty} = 0$, b_n decrease

so converges by A.S.T.

alternating series test

[turns out it converges conditionally, but that wasn't asked!]

b. $\sum_{n=1}^{\infty} \frac{1}{(1+\ln n)^n}$ root test

$$\rho = \lim_{n\to\infty} \sqrt[n]{\frac{1}{(1+\ln n)^n}} = \lim_{n\to\infty} \frac{1}{1+\ln n} = \frac{1}{\infty} = 0 < 1$$

so converges

EC. [1 points] (**Extra Credit**) How close is the partial sum $S_9 = \sum_{n=2}^{9} \frac{(-1)^n}{\ln(n)}$ to the conditionally-convergent series $S = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$? State your reason, and write a specific upper bound on the remainder R_9 in the box.

for alternating series $\sum_{n=1}^{\infty} (-1)^n b_n$ which converge, $|R_N| \le b_{N+1}$. here $N=9$ so

$b_{N+1} = \ln(10)$

$|R_9| \le \boxed{\frac{1}{\ln(10)}}$

[recall: $R_N = \underbrace{\sum_{n=\square}^{\infty} a_n}_{S} - \underbrace{\sum_{n=\square}^{N} a_n}_{S_N}$. the starting index "$\square$" does not matter]

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