### 20 October, 2022

## Name: \_

/ 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

SOLUTIONS

1. [9 points] Compute and simplify the improper integrals, or show that they diverge. Use correct limit notation.

a. 
$$\int_{0}^{\infty} \frac{1}{4+x^{2}} dx = \lim_{t \to \infty} \frac{1}{2} \arctan\left(\frac{x}{2}\right) \int_{0}^{t} \frac{1}{2} \left( \arctan\left(\frac{x}{2}\right) \right) = \lim_{t \to \infty} \frac{1}{2} \left( \arctan\left(\frac{x}{2}\right) - 0 \right)$$
$$= \frac{1}{2} \cdot \frac{\pi}{2} = \left(\frac{\pi}{4}\right)$$
$$\begin{bmatrix} or \ by \ trig. \ subst.: \ x = 2 \tan \theta, \ dx = 2 \sec^{2} \theta d\theta \\ g_{1} \sin \theta & \int_{0}^{a \cosh(4x)} \frac{2 \sec^{2} \theta d\theta}{4 \sec^{2} \theta} = \dots = \frac{\pi}{4} \end{bmatrix}$$
  
b. 
$$\int_{0}^{\infty} \sin x dx = \lim_{t \to \infty} \int_{0}^{t} \sin x dx = \lim_{t \to \infty} \left[ -\cos x \right]_{0}^{t}$$
$$= \lim_{t \to \infty} \left[ -\cos t \right] \left( \frac{d \cdot n \cdot e}{2} \right)$$

$$c. \int_{0}^{1} \frac{1}{\sqrt[4]{x}} dx = \lim_{t \to 0^{+}} \int_{t}^{1} x^{-1/4} dx = \lim_{t \to 0^{+}} \frac{4}{3} x^{-1/4}$$

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**2.** [4 points] Verify that  $y = e^{x^2/2}$  solves the differential equation y' = xy.

$$y' \stackrel{\cdot}{=} x y$$
  
 $e^{x^2/2}(x) = x e^{x^2/2}$ 

**3.** [4 points] Find the general solution to the differential equation  $y' = \ln x + \tan x$ .

$$y(x) = \int hx dx + \int ton x dx$$

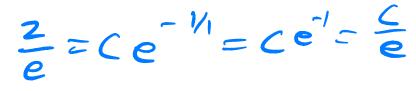
$$= x hx - \int x \cdot \frac{1}{x} dx + \int \frac{-du}{u} -du = s h x dx$$

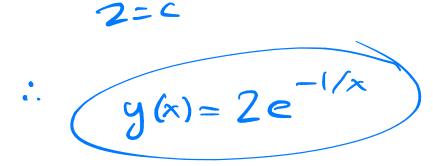
$$= x hx - \int x \cdot \frac{1}{x} dx + \int \frac{-du}{u} = \frac{1}{u} \int \frac{1}{$$

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4. [4 points] Find the particular solution of the differential equation  $y' = \frac{y}{x^2}$  that passes through  $\left(1, \frac{2}{e}\right)$ , given that  $y = Ce^{-1/x}$  is the general solution.

y=ce-Vx at x=1, y==





5. [4 points] Find the area of the region in the first quadrant between the curve  $y = e^{-3x}$  and the *x*-axis.

 $A = \int_{0}^{\infty} e^{-3x} dx = \lim_{t \to \infty} \int_{0}^{t} e^{-3x} dx$  $= \lim_{t \to \infty} \left[ \frac{e^{-3x}}{-3} \right]_{0}^{t} = \frac{1}{3} \lim_{t \to \infty} \left( 1 - e^{-3t} \right)$  $=\frac{1}{2}(1-0)=$ 

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**EC.** [1 points] (Extra Credit) Suppose the curve  $y = e^{-x}$ , on the interval  $[0, +\infty)$ , is rotated around the *x*-axis. Set up an integral for the surface area and give a convincing argument that this integral is finite.