Math 252: Quiz 5 **29 September, 2022**

Name: SOLUTIONS

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30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [4 points] Compute and simplify the indefinite integral:

$$\int \sin^3\theta\cos^3\theta\,d\theta = \int \sin^3\theta\cos^2\theta\cdot\cos\theta\,d\theta$$

$\left[\begin{array}{l} u=\sin\theta \\ du=\cos\theta\,d\theta \end{array}\right]$

$$= \int \sin^3\theta(1-\sin^2\theta)\cdot\cos\theta\,d\theta$$

$$= \int u^3(1-u^2)\,du = \int u^3-u^5\,du = \frac{u^4}{4}-\frac{u^6}{6}+C$$

$$= \frac{1}{4}\sin^4\theta-\frac{1}{6}\sin^6\theta+C$$

or:

$$= \int (1-\cos^2\theta)\cos^3\theta\cdot\sin\theta\,d\theta$$

$\left[\begin{array}{l} u=\cos\theta \\ -du=\sin\theta\,d\theta \end{array}\right]$

$$= \int(1-u^2)u^3(-du) = \frac{u^6}{6}-\frac{u^4}{4}+C = \frac{1}{6}\cos^6\theta-\frac{1}{4}\cos^4\theta+C$$

2. [4 points] Compute and simplify the definite integral:

$$\int_{-2}^{0} xe^x\,dx = xe^x\Big]_{-2}^{0} - \int_{-2}^{0} e^x\,dx$$

$\left[\begin{array}{ll} u=x & v=e^x \\ du=dx & dv=e^x dx \end{array}\right]$

$$= 0-(-2)e^{-2}-\left[e^x\right]_{-2}^{0}$$

$$= +2e^{-2}-1+e^{-2} = \frac{3}{e^2}-1$$

3. [5 points] Find the area of the region bounded by $y = e^x \sin x$ and the x-axis, on the interval $0 \le x \le \pi$.

$$A = \int_0^\pi e^x \sin x\,dx = e^x(-\cos x)\Big]_0^\pi - \int_0^\pi (-\cos x)e^x\,dx$$

$$\begin{bmatrix} u = e^x & v = -\cos x \\ du = e^x dx & dv = \sin x\,dx \end{bmatrix}$$

$$= e^\pi(+1) - e^0(-1) + \int_0^\pi e^x \cos x\,dx$$

$$= e^\pi + 1 + \left(e^x \sin x\Big]_0^\pi - \int_0^\pi e^x \sin x\,dx \right)$$

$$\begin{bmatrix} w = e^x & z = \sin x \\ dw = e^x dx & dz = \cos x\,dx \end{bmatrix}$$

$$= e^\pi + 1 + (0 - 0 - A)$$

so $A = e^\pi + 1 - A$ so $2A = e^\pi + 1$

so $A = \frac{1}{2}(e^\pi + 1)$

4. [4 points] Compute and simplify the indefinite integral:

$$\int t^3 \ln t\,dt = \frac{1}{4}t^4 \ln t - \int \frac{1}{4}t^4 \cdot \frac{1}{t}\,dt$$

$$\begin{bmatrix} u = \ln t & v = \frac{1}{4}t^4 \\ du = \frac{1}{t}dt & dv = t^3 dt \end{bmatrix}$$

$$= \frac{1}{4}t^4 \ln t - \frac{1}{4}\int t^3\,dt = \frac{1}{4}t^4 \ln t - \frac{t^4}{16} + C$$

$$= \frac{t^4}{4}\left(\ln t - \frac{1}{4}\right) + C$$

5. [4 points] Compute and simplify the indefinite integral. (*Hint. You may have this integral memorized, but I have asked you to remember the trick which does it. So please apply the trick!*)

$$\int \sec x\,dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x}\,dx$$

$$= \int \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}\,dx$$

$u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x)\,dx$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

6. [4 points] Compute and simplify the indefinite integral:

$$\int \cos^2 x \sin^2 x\,dx = \frac{1}{4}\int (1+\cos(2x))(1-\cos(2x))\,dx$$

$$= \frac{1}{4}\int 1 - \cos^2(2x)\,dx$$

$$= \frac{1}{4}\left[x - \frac{1}{2}\int 1 + \cos(4x)\,dx\right]$$

$$= \frac{x}{4} - \frac{1}{8}\left(x + \frac{\sin(4x)}{4}\right) + C$$

$$= \frac{x}{8} - \frac{1}{32}\sin(4x) + C = \frac{1}{32}(4x - \sin(4x)) + C$$

EC. [1 points] (**Extra Credit**) Assume n is a large integer. One of these indefinite integrals is much easier than the other. Circle the easier one, and do it.

$$\int \sec^n x \tan x \, dx \qquad\qquad \int \tan^n x \sec x \, dx$$

$$= \int \sec^{n-1} x \sec x \tan x \, dx$$

$$\left[\begin{array}{l} u = \sec x \\ du = \sec x \tan x \, dx \end{array} \right]$$

$$= \int u^{n-1} \, du$$

$$= \frac{u^n}{n} + C = \frac{\sec^n x}{n} + C$$

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