Math 252: Quiz 3 SOLUTIONS **15 September, 2022**

Name: ______________________ ________ **/ 25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [9 points] For each part below, **completely set up**, **but do not evaluate**, an integral for the quantity.

a. The length of the curve $y = \frac{x^2}{8} - \ln x$ on the interval $1 \le x \le 3$.

$f(x) = \frac{x^2}{8} - \ln x$

$f'(x) = \frac{x}{4} - \frac{1}{x}$

$$L = \int_1^3 \sqrt{1 + \left(\frac{x}{4} - \frac{1}{x}\right)^2}\, dx$$

b. The area of the surface formed by revolving the graph of $y = x^4$, on the interval $[-1, 1]$, around the x-axis.

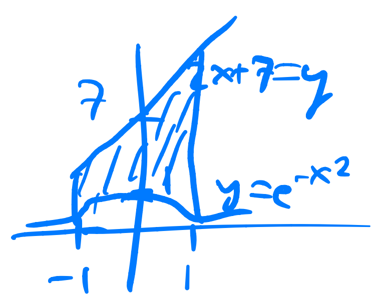
$f(x) = x^4$

$f'(x) = 4x^3$

$$A = \int_{-1}^{1} 2\pi x^4 \sqrt{1 + (4x^3)^2}\, dx$$

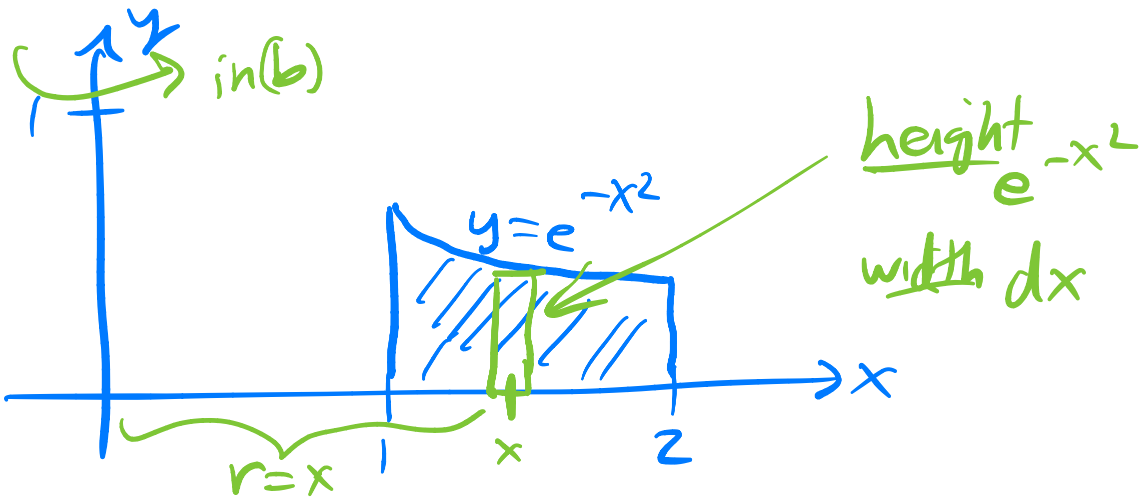
c. The area between the graphs of $y = e^{-x^2}$ and $y = 2x + 7$ on the interval $[-1, 1]$.

$$A = \int_{-1}^{1} (2x+7) - e^{-x^2}\, dx$$



2. [8 points]

a. Sketch the region between $y = e^{-x^2}$ and the x-axis, on the interval $[1,2]$.



b. Find the volume of the solid formed by revolving the region in part **a** around the y-axis. (*Yes, you can do the integral if you use the right volume technique.*)

$$V = \int_1^2 2\pi x \cdot e^{-x^2} \cdot dx$$

$$u = -x^2$$
$$du = -2x\,dx$$
$$\frac{du}{-2} = x\,dx$$

$$= 2\pi \int_{-1}^{-4} e^u \frac{du}{-2}$$

$$= \pi \int_{-4}^{-1} e^u \, du$$

$$= \pi \left(e^{-1} - e^{-4}\right)$$

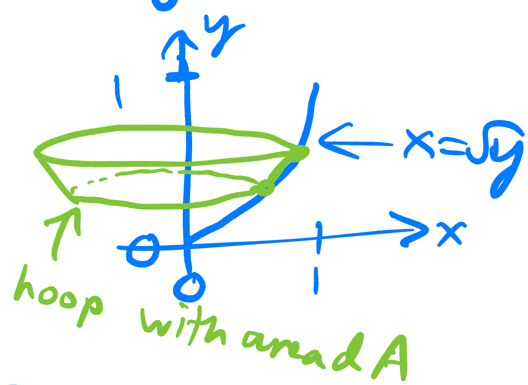
3. [8 points] Find the area of the surface of revolution from rotating $y = x^2$ from $x = 0$ to $x = 1$ around the y-axis. (*Yes, you can do the integral.*)

$$A = \int_0^1 2\pi\sqrt{y}\sqrt{1+\left(\frac{1}{2\sqrt{y}}\right)^2}\,dy$$

$$g(y) = \sqrt{y}$$

$$g'(y) = \frac{1}{2}y^{-1/2}$$

$$= 2\pi\int_0^1 \sqrt{y}\sqrt{1+\frac{1}{4y}}\,dy$$



$$= 2\pi\int_0^1 \sqrt{y+\tfrac{1}{4}}\,dy$$

$$= 2\pi\int_{1/4}^{5/4}\sqrt{u}\,du \qquad \left[\begin{array}{l} u = y+\frac{1}{4} \\ du = dy \end{array}\right]$$

$$= 2\pi\cdot\frac{2}{3}\,u^{3/2}\Big]_{1/4}^{5/4}$$

$$= \frac{4\pi}{3}\left(\left(\tfrac{5}{4}\right)^{3/2} - \left(\tfrac{1}{4}\right)^{3/2}\right) \Leftarrow \text{just fine!}$$

$$= \frac{4\pi}{3}\left(\frac{5^{3/2}-1}{8}\right) \qquad \left(\text{since } 4^{3/2} = \left(4^{1/2}\right)^3\right)$$

$$= \frac{\pi}{6}\left(5^{3/2}-1\right) \Leftarrow \text{also just fine!}$$

EC. **[1 points]** (**Extra Credit**) Though I do not know how to find antiderivatives for the integrals in **1b** and **1c**, the integral in **1a** can be computed by hand. Do so.

$$L = \int_1^3 \sqrt{1+\left(\frac{x}{4}-\frac{1}{x}\right)^2}\,dx$$

$$= \int_1^3 \sqrt{1+\frac{x^2}{16}-\frac{1}{2}+\frac{1}{x^2}}\,dx = \int_1^3 \sqrt{\frac{x^2}{16}+\frac{1}{2}+\frac{1}{x^2}}\,dx$$

$$= \int_1^3 \sqrt{\left(\frac{x}{4}+\frac{1}{x}\right)^2}\,dx = \int_1^3 \frac{x}{4}+\frac{1}{x}\,dx$$

$$= \frac{x^2}{8}+\ln|x|\Big]_1^3 = \frac{9}{8}+\ln(3)-\left(\frac{1}{8}+0\right) = 1+\ln(3)$$

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