Name:______________________________________

Math 252 Calculus 2 (Bueler) Thursday, 17 February 2022

Midterm Exam 1

No book, notes, electronics, calculator, or internet access. 100 points possible. 70 minutes maximum.

1. (*8 pts*) Compute the area between the curves $y = \cos(x)$ and $y = \cos^2(x)$ on the interval $0 \le x \le \pi/2$. (*Hint. Be careful about which curve is above the other.*)

2. (*6 pts*) Completely set up, but do not evaluate, a definite integral for the length of the curve $y = \dfrac{1}{x}$ on the interval $x = 1$ to $x = 10$.

3. Evaluate and simplify the following indefinite and definite integrals.

(a) (*6 pts*) $\displaystyle\int \tan x \, dx =$

(b) (*6 pts*) $\displaystyle\int_0^1 3^x \, dx =$

(c) (*6 pts*) $\displaystyle\int_0^{\pi/4} \tan^3 x \sec^2 x \, dx =$

(d) (*8 pts*) $\displaystyle\int \cos^2(7t)\sin^3(7t)\,dt =$

(e) (*8 pts*) $\displaystyle\int \cos(7t)\cos(3t)\,dt =$

(f) (*8 pts*) $$\int \frac{x}{x^2 - 4x - 5}\, dx =$$

(g) (*8 pts*) $$\int z^2\, e^z\, dz =$$

4. (a) (*4 pts*) Sketch the region bounded by the curves $y = \ln x$, $x = 1$, and $y = 1$.

(b) (*8 pts*) Use shells to find the volume of the solid of revolution found by rotating the region in part **(a)** around the x-axis.

(c) (*8 pts*) Fully set up, but do not evaluate, the three integrals needed to compute the center of mass $(\bar{x}, \bar{y})$ of the region in part **(a)** (previous page). Then fill in the blanks at the bottom, to show how to compute the values $\bar{x}$ and $\bar{y}$.

$m =$

$M_y =$

$M_x =$

$$\bar{x} = \frac{\boxed{}}{\boxed{}}, \qquad \bar{y} = \frac{\boxed{}}{\boxed{}}$$

5. Which trigonometric substitution would you use for the following two integrals? Write the substitution in the box. (There is no need to compute the integrals here.)

(a) (*4 pts*) $\displaystyle\int \sqrt{x^2 - 16}\, dx$

(b) (*4 pts*) $\displaystyle\int \frac{t^2}{\sqrt{1 - 4t^2}}\, dt$

6. (*8 pts*) Evaluate and simplify the integral in **5(b)** above.

Extra Credit. (*3 pts*) A donut (torus) surface is created by rotating a circle with radius one and center $(x, y) = (2, 0)$ around the y-axis. Fully set up, but do not evaluate, an integral for the surface area of this donut.

You may find the following **trigonometric formulas** useful. However, there are other trig. formulas, not listed here, which you should have in memory, or which you know how to derive from these.

$$\begin{aligned}
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\
\sin(ax)\sin(bx) &= \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x) \\
\sin(ax)\cos(bx) &= \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x) \\
\cos(ax)\cos(bx) &= \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)
\end{aligned}$$