Name:

Math 252 Calculus 2 (Bueler)

Thursday, 17 February 2022

## Midterm Exam 1

## No book, notes, electronics, calculator, or internet access. 100 points possible. 70 minutes maximum.

**1.** (8 pts) Compute the area between the curves  $y = \cos(x)$  and  $y = \cos^2(x)$  on the interval  $0 \le x \le \pi/2$ . (*Hint. Be careful about which curve is above the other.*)

**2.** (6 pts) Completely set up, but do not evaluate, a definite integral for the length of the curve  $y = \frac{1}{x}$  on the interval x = 1 to x = 10.

**3.** Evaluate and simplify the following indefinite and definite integrals.

(a) 
$$(6 \ pts)$$
  $\int \tan x \, dx =$ 

**(b)** 
$$(6 \ pts) \qquad \int_0^1 3^x \, dx =$$

(c) 
$$(6 \ pts) = \int_0^{\pi/4} \tan^3 x \sec^2 x \, dx =$$

(d)  $(8 \ pts) \int \cos^2(7t) \sin^3(7t) dt =$ 

(e)  $(8 \ pts) \int \cos(7t) \cos(3t) dt =$ 

(f) (8 pts) 
$$\int \frac{x}{x^2 - 4x - 5} dx =$$

(g) (8 pts) 
$$\int z^2 e^z dz =$$

4. (a) (4 *pts*) Sketch the region bounded by the curves  $y = \ln x$ , x = 1, and y = 1.

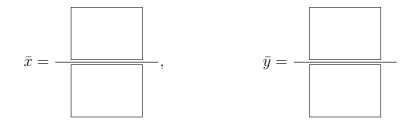
(b)  $(8 \ pts)$  Use shells to find the volume of the solid of revolution found by rotating the region in part (a) around the x-axis.

(c)  $(8 \ pts)$  Fully set up, but do not evaluate, the three integrals needed to compute the center of mass  $(\bar{x}, \bar{y})$  of the region in part (a) (previous page). Then fill in the blanks at the bottom, to show how to compute the values  $\bar{x}$  and  $\bar{y}$ .

m =

 $M_y =$ 

 $M_x =$ 



5. Which trigonometric substitution would you use for the following two integrals? Write the substitution in the box. (There is no need to compute the integrals here.)

(a) 
$$(4 \ pts) = \int \sqrt{x^2 - 16} \ dx$$

**(b)** 
$$(4 \ pts) = \int \frac{t^2}{\sqrt{1-4t^2}} dt$$



**6.**  $(8 \ pts)$  Evaluate and simplify the integral in **5(b)** above.

**Extra Credit.** (3 pts) A donut (torus) surface is created by rotating a circle with radius one and center (x, y) = (2, 0) around the y-axis. Fully set up, but do not evaluate, an integral for the surface area of this donut.

You may find the following **trigonometric formulas** useful. However, there are other trig. formulas, not listed here, which you should have in memory, or which you know how to derive from these.

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$
$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$
$$\sin(ax) \sin(bx) = \frac{1}{2} \cos((a-b)x) - \frac{1}{2} \cos((a+b)x)$$
$$\sin(ax) \cos(bx) = \frac{1}{2} \sin((a-b)x) + \frac{1}{2} \sin((a+b)x)$$
$$\cos(ax) \cos(bx) = \frac{1}{2} \cos((a-b)x) + \frac{1}{2} \cos((a+b)x)$$