Your Name

Signature (you agree to complete honestly)

Student ID #

Start Time

End Time

Page	Total Points	Score
2	10	
3	10	
4	11	
5	13	
6	10	
7	8	
8	7	
9	8	
10	10	
11	10	
12	3	
Total	100	

- You will have 2.5 hours to complete the exam.
- This test is closed book and you may not use a calculator.
- You may use one side of a single piece of paper (8 1/2 in. x 11 in.) of handwritten notes.
- In order to receive full credit (or partial credit in the case of incorrect solutions), you must **show your work.** Please write out your computations on the exam paper.
- Simplify all obvious expressions.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.

1. (20 points) Evaluate the following integrals.

(a) $\displaystyle\int_0^{\pi/4} \sec^4 x \tan^2 x \, dx$

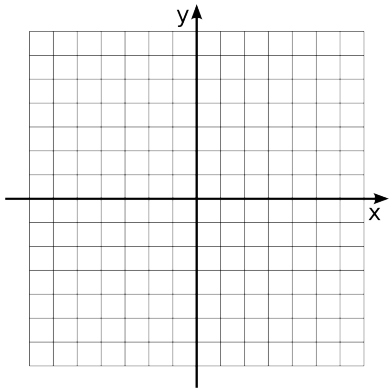
(b) $\displaystyle\int \arctan(2x) \, dx$

(c) $\displaystyle\int \frac{4x}{(x^2+4)(x-2)}\,dx$

(d) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$

2. (11 points) Let R be the region bounded by the graphs of $f(x) = 2x - 1, \quad g(x) = (x-2)^2$.

(a) Graph the region and then set up, but do not solve, an integral that gives the **area** of R.



(b) Set up, but do not solve, an integral that finds the **volume** of the solid when R is rotated about the y-axis.

(c) Set up, but do not solve, an integral that finds the volume of the solid when R is rotated about the line $y = -1$.

(d) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square whose sides are the length of the base in the region R. Set up, but do not solve, an integral that gives the volume of this solid.

3. (4 points) Let $a_n = \sin\left(\dfrac{n - \pi n^2}{2n^2 + 3}\right)$.

   (a) Determine whether the sequence a_n converges. If it is convergent determine what it converges to.

   (b) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges. Justify your answer.

4. (5 points) Find the sum of the following series exactly.

   a) $\displaystyle\sum_{n=1}^{\infty} (-2)^n 2^{-2n+1}$

   b) $\displaystyle\sum_{n=0}^{\infty} \frac{3(-1)^n 2^n}{n!}$

5. (4 points) Find the Taylor series for the function $f(x) = e^{3x}$ centered at the point $a = -1$. Give your answer in summation notation.

6. (7 points)

   (a) Determine whether the improper integral $\int_1^\infty xe^{-x^2}\,dx$ converges or diverges. Evaluate it if it is convergent.

   (b) Use the integral test, and your answer from (a), determine whether $\sum_{n=1}^{\infty} ne^{-n^2}$ converges or diverges. You must explicitly verify that the integral test applies to this series. No credit will be given if another test is used.

7. (3 points) Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

   (a) Find s_4. No need to simplify.

   (b) At most, how far is this from the actual sum? I.e., what is the |error|?

8. (8 points) Determine whether the following series converge or diverge. You must clearly explain your reasoning.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+3}}{4+2n^3}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n+3}$

9. (7 points) Find the center, radius of convergence, and the interval of convergence of the following series.

(a) $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{(n+1)^n}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{3^n \cdot n^2}$

10. (8 points) Let $\mathcal{R}$ be the region bounded by $y = e^x$ and $y = 0$, $0 \leq x \leq 1$.

    (a) Sketch the region and find the area of $\mathcal{R}$.

    (b) Find the centroid of the the region $\mathcal{R}$.

11. (10 points) Consider $x = t + 2\ln t, \quad y = t - \ln t.$

(a) Find and simplify $\dfrac{dy}{dx}$.

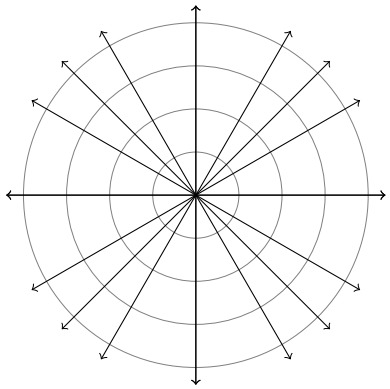
(b) Determine the location of any horizontal tangents. If none exist, explain why.

(c) Find and simplify $\dfrac{d^2y}{dx^2}$.

(d) Determine the values of t for which the curve is concave up.

12. (10 points) Consider the curve $r = 2\sin(3\theta)$.

(a) Sketch the curve $r = 2\sin(3\theta)$.

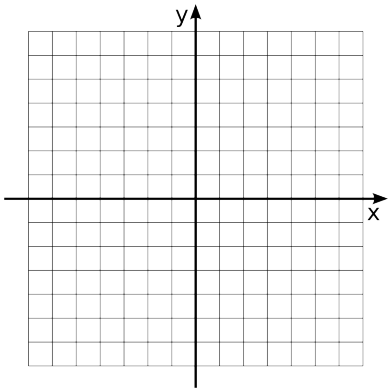


(b) Find the area enclosed by one petal.

(c) Set up, but do not solve, an integral that gives the length of the polar curve traced out once.

13. (3 points) Consider the curve defined by the parametric equations $x = e^t, \quad y = 2e^{-t}$.

(a) Graph the curve and indicate with an arrow the direction in which the curve is traced as t increases and (b) eliminate the parameter to find a Cartesian equation of the curve. [Make sure to specify any restriction on the variables.]



Extra Credit (3 points) Find the length of the polar curve $r = e^{-\theta}$ for $\theta \geq 0$.