Math 252 Exam 3 | Name:____________________________________

11/18/2020 | Instructor: Rhodes

Show all your work.

1. (20 pts.) A function $f(x)$ is defined by the series

$$f(x) = \sum_{n=0}^{\infty} \frac{5^n}{(n+1)2^n}(x-3)^n.$$

(a) (5 pts.) Determine the radius of convergence of the series.

(b) (6 pts.) Determine the interval of convergence of the series.

(c) (5 pts.) Give a series for $\int f(x)\,dx$.

(d) (4 pts.) What is $f^{(9)}(3)$?

2. (14 pts.) Using the definition, find the Taylor series for the function $f(x) = \cos x$ at $a = \dfrac{\pi}{4}$. Write out at least the first 5 terms.

3. (14 pts. – 7 pts. each) Working from power series you already know, calculate at least the first 3 non-zero terms power series for the following.

   (a) $\dfrac{1}{4+x}$

   (b) $\dfrac{\sin x - x}{x^2}$

4. (24 pts. – 6 pts. each) Determine whether the following series converge or diverge. For each, *state what test you use*, and write enough to indicate that you checked all conditions necessary to apply the test.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^3+7n+1}{n^6-5n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3+2}{5n^3+n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$

5. (12 pts. – 6 pts. each) For the series

$$\sum_{n=0}^{\infty} \frac{2}{(2n+5)(2n+7)} = \sum_{n=0}^{\infty} \left(\frac{1}{2n+5} - \frac{1}{2n+7} \right),$$

(a) Find a formula for the nth partial sum, s_n.

(b) Using s_n, determine whether the series converges, and if it does, determine its limit.

6. (14 pts. – 7 pts. each)

(a) Use the integral test to show the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges.

(b) The remainder estimate for the integral test says that if the series converges to s, then the difference $R_n = s - s_n$ between the series and its nth partial sum satisfies

$$\int_{n+1}^{\infty} f(x)\,dx \le R_n \le \int_{n}^{\infty} f(x)\,dx.$$

Use this to determine how many terms of $\sum_{n=1}^{\infty} \frac{1}{n^4}$ need to be added up to estimate s to within 10^{-10}