Consequences of the Baire theorem

XXX YYY

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the Baire theorem

Definition

if \((X, d)\) is a metric space and \(S \subseteq X\) then we say \(S\) is *nowhere dense* if the closure \(\overline{S}\) contains no (positive radius) balls

- equivalently, the interior of the closure \((\overline{S})^\circ\) is empty

Theorem (Baire 1899)

*a nonempty complete metric space is not a countable union of nowhere dense sets*

- that is, if \((X, d)\) is complete and \(X = \bigcup_{n=1}^{\infty} A_n\) then there exists a subset \(A_n\) whose closure contains a ball: \(B_r(x) \subseteq \overline{A_n}\) for \(x \in X\) and \(r > 0\)
- the proof of the Baire theorem requires the axiom of choice
IT IS OFTEN GOOD TO USE ITEMS FOR SLIDES