Assignment #9

Due Friday, 24 April 2020, at 5pm. Send electronically.

Please read Chapter 15 on C^*-algebras. Regarding these abstract algebras, note that you may restrict to bounded operators on Hilbert spaces H without losing any meaning at all. That is, you may assume that a C^*-algebra $\mathcal{A}$ is actually $\mathcal{A} = B(H)$.

Read the material through the spectral theorem for compact normal operators (Theorem 15.23), the singular value decomposition for compact operators, and the approximation theorem for compact operators (Proposition 15.26).

The material after Proposition 15.26, and certain material before that, is either unimportant or presented poorly (in my humble opinion). Also, I will give a separate presentation of the spectral theorem for normal operators (i.e. for non-compact operators). Thus, you may **skip** discussion of state spaces $S(T)$, numerical range $W(T)$, Fuglede's theorem, and anything after Proposition 15.26.

This Assignment is shorter than it looks because several exercises yield to quick calculations.

One exercise below is identified with your initials. Please LaTeX this problem and send both the `.tex` and `.pdf` to me at `elbueler@alaska.edu` by the due date.

DO THE FOLLOWING EXERCISES from the textbook (Muscat, *Functional Analysis*, 2014):

- #4 in Exercises 15.11, page 352.

For the next three exercises, recall Proposition 10.23 describes unitary operators.

- #7 in Exercises 15.11, page 352. ⟵ **DD** *Note the distinction between $\ell^2(\mathbb{Z})$ and $\ell^2 = \ell^2(\mathbb{N})$.*
- #8 in Exercises 15.11, page 352. ⟵ **OS**
- #9 in Exercises 15.11, page 352. ⟵ **WV**

- #1 in Exercises 15.15, page 354.
- #2 in Exercises 15.15, page 354.
- #4 in Exercises 15.15, page 354.
- #5 in Exercises 15.15, page 354.
- #6 in Exercises 15.15, page 354.