Project B worksheet: a periodic ODE

ONLY TWO RULES:

1. DON'T USE THE INTERNET
2. DO TALK TO EACH OTHER

(ON 2., TRY TO *both* SHARE UNDERSTANDINGS *and* DIVIDE-UP TASKS.)

A compressible fluid flows with known velocity $v(x) = 30(1 + \sin x)$ through a thin tube with length 2π, formed into a closed circle of tubing with no outlet. The fluid has a steady state temperature $u(x)$ determined by this equation:

$$u''(x) - v(x)u'(x) + 5u(x) = -f(x) \quad \text{on } 0 < x \le 2\pi,$$

where x is the distance along the tube from some location. (*The equation is a balance of diffusion (u'' term), motion (vu' term) and reaction proportional to the temperature (u term), along with an internal heat source (f).*) Propose and implement, in MATLAB/etc., a verified FD scheme for this problem.

(*Hint.* Be careful when drawing the grid and deciding on the locations of unknowns. Is there a boundary, and should there be boundary conditions? Note the problem may be hard to integrate (solve) by hand, but doing so is *not* the only way to find an exact solution for verification purposes.)