

Assignment #5

Due Monday 11 October, 2021 at the start of class

Please read Lectures 6, 7, 8, 9, and 10 in the textbook *Numerical Linear Algebra* by Trefethen and Bau. Then do the following exercises.

- P12.** (a) Give an example of a projector which is *not* an orthogonal projector.
(b) Show that if P is a projector and λ is an eigenvalue of P then $\lambda = 0$ or $\lambda = 1$.
(c) Show that if a projector is invertible then it is the identity.
(d) Show that if $\lambda = 1$ is *not* an eigenvalue of a projector P then in fact $P = 0$.

P13. Just for fun, write a Matlab/Octave (or other) function

```
function P = randproj(m,k)
```

which generates a random orthogonal projection $P \in \mathbb{R}^{m \times m}$ with $\text{rank}(P) = k$. Note k is an integer in the range $0 \leq k \leq m$. (*Hint.* Perhaps start by generating a random *unitary* matrix Q by using the SVD, e.g. `[Q, S, V]=svd(randn(n,n)).`) Verify the function by checking that the output is an orthogonal projector and that it has the desired rank.

Exercise 5.4.

Exercise 6.4.

Exercise 7.1. (“any method you like” means any by hand method you like!)

Exercise 7.3.

Exercise 8.1.

Exercise 8.2.