Assignment #4

Due Monday 27 September, 2021 at the start of class

Please read Lectures 4, 5, and 6 in the textbook *Numerical Linear Algebra* by Trefethen and Bau. Then do the following exercises.

P10. Use by-hand calculations to determine SVDs of the following matrices. Note that in the decomposition $A = U\Sigma V^*$, the factor Σ is unique but the factors U, V are not, and thus there will be more than one correct answer. (*Hints. First, think. Then use Theorem 5.4 or Theorem 5.5 if needed. When in doubt, check (and show) that the unitary factors are indeed unitary.*)

(a) $\begin{bmatrix} 5 & 0 \\ 0 & -3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3 & 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

P11. *Theorem 5.7 interprets the SVD as saying that any matrix A is the sum of rank-one matrices, a sum of outer products from columns of U and V, and weighted by the singular values σ_j. Furthermore Theorem 5.8 shows that the partial sums are the nearest lower-rank approximations to A in the induced 2-norm. The goal of this exercise is to* see *these ideas.*

(a) Consider the 4×3 matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \\ 2 & 6 & 5 \\ 3 & 5 & 8 \end{bmatrix}$$

Using Matlab/Octave to compute the full SVD of A. What is the rank of A? (*This time, please show the gory details of the factors U, Σ, V. When you show these matrices, round all entries at the 3rd place past the decimal, for relative brevity.*)

(b) Part **(a)** suggests that printing the entries of generic unitary matrices is not very informative. On the other hand we can view matrices as images. In fact, save the following function, also posted at the public course site, as `blockimage.m`:

```
                        ┌─ blockimage.m ─┐
function blockimage(A)
% BLOCKIMAGE   Generate a monochrome block image of a matrix
% using IMAGESC.  Example:
%    >> blockimage(magic(5))

colormap('gray')        % make a monochrome image
imagesc(A)              % treat A as an image (scaled shading)
axis equal, axis off  % each entry is a square; declutter
```

Now generate side-by-side block images of the matrix A in part **(a)**, and its rank-two and rank-one best approximations.

(c)   Describe in words what the block image of any rank-one matrix looks like.

(*Hints on part* **(c)**: Give two or three sentences of artistic commentary. But first, feel free to look at more examples as needed. I generated random matrices `A = randn(20,15)` and then compared `blockimage(A)` to `blockimage(A1)` where `A1` was the rank-one approximation to `A`, i.e. from Theorem 5.8. Recall that a rank-one matrix can always be written as a single outer product.)

Exercise 3.4.

Exercise 4.4.

Exercise 5.1.

Exercise 5.2.