Assignment #9

Due Wednesday 17 November, 2021 at the start of class.

Submit on paper or by email: `elbueler@alaska.edu`

Exercise 5.6.1. Do part (e) only. Explain why we *do not* get 2nd-order.
(*Hint. What is* $M = \|f''\|_\infty$ *in this case?*)

Exercise 5.6.4.

Exercise 5.6.6. Do parts (a) and (c) only.

P9. (*This is a preview of short-answer questions which might appear on the Final Exam.*)

Recall that in Chapter 5 we studied three interpolation schemes: **polynomial interpolation**, **piecewise-linear (PL) interpolation**, and **cubic-spline interpolation**. For each of the following questions, give an answer in a sentence or two.

- **(a)** Suppose we have $n+1$ points (t_i, y_i) with $t_0 < t_1 < \dots < t_n$. For each of the three interpolation schemes, how many unknowns need to be determined to compute the interpolant?
- **(b)** Of the three interpolation schemes, which require solving linear systems?
- **(c)** Which of the three interpolation schemes generally has the best conditioning? Which generally has the worst?
- **(d)** The cubic spline coefficients solve a long-ish list of equations. State in words, but reasonably precisely, what these equations require.

P10. *This problem replaces Exercise 5.7.1 with an easier goal for a single integral.*

Consider this integral which we did by trapezoid rule in Exercise 5.6.1 (e) above:

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}.$$

Download the Function 5.7.1, `intadapt.m`, and use it on this integral. Specifically:

(a) Generate a plot like the one at the bottom of page 221 using `tol=1e-4`$= 10^{-4}$. How accurate was the result, and how many nodes did `intadapt.m` use? Redo the calculation with `tol=1e-6`; again report accuracy and number of nodes.

(b) Explain in a few sentences what feature of the integrand in this case is difficult (*hint: Exercise 5.6.1(e)*), how this adaptive integration method detects the difficulty, and how the method addresses the difficulty by applying greater effort.

P11. *This by-hand problem replaces and simplifies Exercise 6.1.1.*

For each IVP below, determine whether the problem satisfies the conditions of Theorem 6.1.1. If so, determine the smallest possible value for L.

(a) $f(t, u) = 3u, \quad 0 \le t \le 1, \quad u_0 = -2$

(b) $f(t, u) = -t\sin(u), \quad 0 \le t \le 5, \quad u_0 = 1$

(c) $f(t, u) = -(1 + t^2)u^2, \quad 1 \le t \le 3, \quad u_0 = 3$

P12. *This Matlab problem replaces and corrects Exercise 6.1.2.*

Solve each IVP in the preceding problem with Matlab's built-in function `ode45` using default settings, and make a plot of the solution.