Assignment #5

Due Monday 11 October, 2021 at the start of class.

Submit on paper or by email: `elbueler@alaska.edu`

Exercise 2.6.3. (*Turn in parts* (a) *and* (b) *only. Note the banded matrix (Section 2.9).*)

Exercise 2.7.2. (*Just to emphasize, your argument/proof must apply for* all $x \in \mathbb{R}^n$.)

Exercise 2.7.3. (*Again, argue for* all *vectors* x, y.)

Exercise 2.8.1. (*Note Example 2.8.1 already uses such a Hilbert matrix.*)

Exercise 3.1.3.

Exercise 3.1.6. (*Hint. Since you want to fit* $\tau = cR^\alpha$ *you will want to use* log.)

P5. The Matlab/Octave command `cond()` computes the 2-norm condition number $\kappa(A)$ of a matrix:

$$\texttt{cond(A)} = \kappa(A) = \|A^{-1}\|_2 \|A\|_2.$$

The main idea (Section 2.8) is that if $\kappa(A)$ is large then solving a system $Ax = b$ will produce substantial rounding error. If $\kappa(A) > 10^{15} \approx 1/\epsilon_{\text{machine}}$, for example, then one expects the rounding error when solving $Ax = b$ to be so severe that the computed solution x is useless.

(a) So, what is the typical condition number $\kappa(A)$ of a random 2×2 matrix? Write a program that computes the average value of $\log_{10}(\kappa(A))$ for $N = 1000$ matrices each built as `A=randn(2,2)`, and report the resulting mean. What value of $\kappa(A)$ does this represent? Repeat for $N = 10^4$ and $N = 10^5$.

(b) For the random matrices built as in part **(a)**, $\kappa(A) > 100$ is not that rare, and in fact I find that it occurs more than 1% of the time. (*However,* $\kappa(A) > 10^{15}$ *must be extraordinarily rare.*) Write a program that generates enough random 2×2 matrices `A=randn(2,2)` so that you find one with $\kappa(A) > 100$. Apply the program `bueler.github.io/M426F21/matlab/vismat.m` to this matrix. Describe what you see. Explain why $\kappa(A) > 100$ produces such a picture.