Worksheet: Harmonic conjugates

*In problems **1**–**3**, determine whether the given function is harmonic. If it is, find $v(x, y)$, the harmonic conjugate, so that $u(x, y) + iv(x, y)$ is analytic. (If given "$v(x, y)$" then find $u(x, y)$.)*

1. $u(x, y) = x \sin y - e^x \cos y$

2. $u(x, y) = xy + 3x^2 y - y^3$

3. $v(x, y) = y \cos x \cosh y - x \sin x \sinh y$

4. (*This problem is a reminder that you solved a related problem in Calculus III, reminiscent of harmonic conjugate calculations.*) Is the given vector field $\mathbf{F}(x, y)$ a gradient? If it is, find $\phi(x, y)$ so that $\mathbf{F} = \nabla \phi$:

$$\mathbf{F} = (\cos(xy) - xy \sin(xy)) \, \mathbf{i} - x^2 \sin(xy)\mathbf{j}$$