Note on problem E1 (b) and (c) on Take-home Final Exam

I would like to clarify and correct problem **E1 (b)**. Note that what you are doing is indeed covered in **Lesson 12**. But I ask you to please show all the major steps of the calculation to get to the formula (12.9), in the case where $\phi(x) = e^{-|x|}$. You will get a special case of this formula,

$$u(x,t) = \frac{1}{2\alpha\sqrt{\pi t}} \int_{-\infty}^{\infty} \phi(y)\, e^{-(x-y)^2/4\alpha^2 t}\, dy. \tag{12.9}$$

Now, in order to simplify your result in **E1 (b)**, my hints are:

- split the integral at $y = 0$, and
- use these two formulas to simplify your result, and eliminate the integral:

$$\int_0^{\infty} e^{-\left[(x-y)^2/A+y\right]}\, dy = \frac{\sqrt{\pi A}}{2} e^{A/4} e^{-x} \operatorname{erfc}\left(-\frac{x}{\sqrt{A}} + \frac{\sqrt{A}}{2}\right),$$

$$\int_{-\infty}^{0} e^{-\left[(x-y)^2/A-y\right]}\, dy = \frac{\sqrt{\pi A}}{2} e^{A/4} e^{x} \operatorname{erfc}\left(+\frac{x}{\sqrt{A}} + \frac{\sqrt{A}}{2}\right).$$

  where erfc is called the *complementary error function*. This special function is built-in to MATLAB/OCTAVE, if you want to use it.

Regarding part **(c)**, you do not need to write a program to plot the solution to **(a)** and **(b)** in order to get the right sketches. Rather, you should think somewhat-physically about the differences between the problems stated in the two parts. These different boundary values will give different behavior over short times (especially at the boundaries of the interval $[0, L]$) and at long times (over the whole interval $[0, L]$). Of course the solution to **(b)** does not "know about" the interval $[0, L]$, but the solution can still be plotted on that interval.