Name: ______________________________

Math 253 Calculus III (Bueler) April 27, 2018

Practice Final Exam

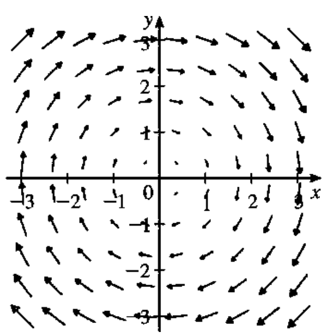
In class. 120 minutes. No calculator. 1 sheet of notes allowed. 200 points total.

- This practice exam is a bit longer than the real thing.
- The problems are generally representative but some sections will not be represented. (We covered too many sections!)
- Here I state the location of the problem in the textbook. On the real exam you must approach each problem without having the "hint" of which section it is from.
- The real exam will give you more space for your answers. Use extra paper here when needed.

1. (§16.1 #21) Find the gradient vector field of

$$f(x,y) = y\sin(xy)$$

2. (§16.2 #17(a)) Let $\mathbf{F}$ be the vector field shown in the figure. If C_1 is the vertical line segment from $(-3,-3)$ to $(-3,3)$, determine whether $\int_{C_1} \mathbf{F}\cdot d\mathbf{r}$ is positive, negative, or zero.



3. (§14.7 #7) Find the local maximum and minimum values and saddle point(s) of the function.

$$f(x,y) = (x-y)(1-xy)$$

4. (§15.4 #9) Find the mass of the lamina that occupies the region D bounded by the curves $y = e^{-x}, y = 0, x = 0, x = 1$ and which has density $\rho(x,y) = xy$.

5. (§15.6 #13) Set up but do not evaluate the triple integral $\iiint_E 6xy\,dV$ if E is under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

6. (§15.8 #21) Use spherical coordinates to evaluate $\iiint_B (x^2+y^2+z^2)^2\,dV$ where B is the ball centered at the origin and with radius 5.

7. (§16.3 #13) **(a)** Find a function f such that $\mathbf{F} = \nabla f$. **(b)** Use part **(a)** to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve.

$$\mathbf{F}(x, y) = x^2y^3\mathbf{i} + x^3y^2\mathbf{j}, \qquad C: \ \mathbf{r}(t) = \left\langle t^3 - 2t, t^3 + 2t \right\rangle, \quad 0 \le t \le 1$$

8. (§16.4 #13) Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F}(x, y) = \langle y - \cos y, x \sin y \rangle$ and C is the circle $(x-3)^2 + (y+4)^2 = 4$ oriented clockwise. (*Note orientation of the curve.*)

9. (§13.2 #31) Find parametric equations for the tangent line to the curve at the given point.

$$x = t\cos t, \quad y = t, \quad z = t \sin t; \qquad (-\pi, \pi, 0)$$

10. (§13.4 #25) A ball is thrown at an angle of 45° to the ground. If the ball lands 90 feet away, what was the initial speed of the ball? (*Use $g = 32\,ft/s^2$ for the acceleration of gravity. Simplify your answer as far as possible without a calculator.*)

11. (§14.3 #49) Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$.

$$e^z = xyz$$

12. (§16.3 #3) Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f so that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y) = (xy + y^2)\mathbf{i} + (x^2 + 2xy)\mathbf{j}$$

13. (§13.3 #7) Set up but do not evaluate an integral which computes the length of the curve.

$$\mathbf{r}(t) = \left\langle t^2, t^3, t^4 \right\rangle, \quad 0 \le t \le 2$$

14. (§12.3 #17) Find the angle between the vectors. (*An exact expression is fine. I know you do not have a calculator.*)

$$\mathbf{a} = \langle 1, -4, 1 \rangle, \quad \mathbf{b} = \langle 0, 2, -2 \rangle$$

15. (§12.5 #31) Find an equation of the plane through the origin and the points $(3, -2, 1)$ and $(1, 1, 1)$.

16. (§15.3 #23) Use polar coordinates in a double integral to find the volume of a sphere of radius a. (*You should know what the answer is, so the work you show is what matters.*)