SOLUTIONS (revised)

Math 252 Calculus II (Bueler) 6 April 2018

Worksheet: Convergence or divergence of series

For each of the following 12 infinite series, state whether it converges or diverges. Justify your statement using the following tests or categories:

- test for divergence
- geometric series
- telescoping series
- p-series
- integral test
- comparison test
- limit comparison test

In many cases multiple tests can determine convergence or divergence.

A. $$\sum_{n=1}^{\infty} \frac{1}{n2^n}$$

Converges — comparison (or limit comparison) to geometric series $\sum_{n=1}^{\infty} \frac{1}{2^n}$

B. $$\sum_{n=1}^{\infty} 2^n$$

diverges — test for divergence ($\lim_{n\to\infty} a_n \neq 0$)

C. $$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

converges — integral test (integration-by-parts) [note: limit comparison not easy like I thought]

D. $$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$$

converges — integral test: use $u=\ln x$ in $\int_2^\infty \frac{1}{x(\ln x)^3}\,dx = \int_{\ln 2}^{\infty} \frac{du}{u^3}$ which converges

E. $$\sum_{n=1}^{\infty} \frac{n-4}{n^3+2n}$$

converges — limit comparison or comparison to $a_n = \frac{1}{n^2}$ (p-series)

F. $$\sum_{n=2}^{\infty} \frac{1+\cos(n)}{e^n}$$

converges — comparison to geometric series with $a_n = \frac{2}{e^n}$

G. $\sum_{n=3}^{\infty} \frac{n^2}{\sqrt{n^3-1}}$ diverges — limit comparison (or comparison) to $a_n = \frac{1}{n}$ (p-series)

H. $\sum_{n=1}^{\infty} \frac{n^3}{(n^4-3)^2}$ converges — limit comparison to $a_n = \frac{1}{n^5}$ (or integral)

I. $\sum_{n=1}^{\infty} (-1)^n 3^{-n/3}$ converges — geometric series with $r = \frac{-1}{3^{1/3}}$

J. $\sum_{n=2}^{\infty} \frac{|\sin(n)|}{n}$ unknown

K. $\sum_{n=2}^{\infty} \frac{1}{n!}$ converges — comparison to $a_n = \frac{1}{2^{n-1}}$ (which converges)

L. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges — limit comparison to $a_n = \frac{1}{n}$ (p-series) (or integral)

M. $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges — limit comparison to $a_n = \frac{1}{n^2}$ (or integral or telescoping)

Finally, some general questions:

(i) In which of the above series can you find the exact sum of the series?

only I and M (and later K)

(ii) In which of the above series could you use a computer to find s_{100}, the sum of the first 100 terms?

all of them.