SOLUTIONS

Math 252 Calculus II (Bueler) 23 April 2018

Worksheet: Power series

1. Find the radius and interval of convergence:

$$\sum_{n=0}^{\infty} \frac{(3x+2)^n}{n!}$$

use ratio test

$$L = \lim_{n\to\infty} \frac{|3x+2|^{n+1}}{(n+1)!}\frac{n!}{|3x+2|^n} = \lim_{n\to\infty} |3x+2| \frac{1}{n+1} = |3x+2|\cdot 0 = 0$$

So $\boxed{R=\infty \text{ and } (-\infty,\infty)}$ is interval

2. Find the radius and interval of convergence:

$$\sum_{n=1}^{\infty} n(x-7)^n$$

use root test (just to spice it up!)

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{n|x-7|^n} = |x-7| \lim_{n\to\infty} \sqrt[n]{n}$$

$$= |x-7|\cdot 1 = |x-7| < 1 \iff -1 < x-7 < 1 \quad 6 < x < 8$$

$x=6$: $\sum_{n=1}^{\infty} n(-1)^n$ diverges

$x=8$: $\sum_{n=1}^{\infty} n$ diverges

$\boxed{R=1 \text{ and } (6,8)}$ is interval of conv.

3. Find the radius and interval of convergence:

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{3^n}$$

use ratio test [in fact this series is geometric!]

$$L = \lim_{n\to\infty} \frac{|x|^{2(n+1)+1}}{3^{n+1}}\frac{3^n}{|x|^{2n+1}} = \lim_{n\to\infty} \frac{|x|^2}{3} = \frac{|x|^2}{3} < 1 \iff -\sqrt{3} < x < \sqrt{3}$$

so $\boxed{R=\sqrt{3}}$

$x=-\sqrt{3}$: $\sum_{n=0}^{\infty} (-1)^n \frac{(-\sqrt{3})^{2n+1}}{3^n} = \sum_{n=0}^{\infty} (-1)^n \frac{(-1)^{2n} 3^n (-1)\sqrt{3}}{3^n} = \sum_{n=0}^{\infty} -(-1)^n\sqrt{3}$ diverges

$x=+\sqrt{3}$: $\sum_{n=0}^{\infty} (-1)^n \frac{(\sqrt{3})^{2n+1}}{3^n} = \sum_{n=0}^{\infty} (-1)^n\sqrt{3}$ diverges

so interval is $\boxed{(-\sqrt{3},\sqrt{3})}$

4. The goal here is to accurately do an integral, by using power series, that we could not do before.

(a) Compute the sum of the series assuming $|x| < 1$:

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

(b) Substitute $-x^4$ for x to get a power series for this function:

$$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)} = 1 - x^4 + x^8 - x^{12} + x^{16} - \cdots$$

(c) Integrate term-by-term to get a power series:

$$\int \frac{1}{1+x^4}\,dx = C + x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13} + \frac{x^{17}}{17} - \cdots$$

(d) What is the radius and interval of convergence of the above series?

$R=1$ and interval is $[-1,1]$

$x=-1$: $C - 1 + \frac{1}{5} - \frac{1}{9} + \frac{1}{13} - \cdots$ converges by Alt. Series

$x=1$: $C + 1 - \frac{1}{5} + \frac{1}{9} - \cdots$ same

(e) Evaluate to get a series (*note x is gone so it is no longer a power series!*):

$$\int_0^{0.2} \frac{1}{1+x^4}\,dx = \left[x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13} + \cdots\right]_0^{0.2}$$

$$= 0.2 - \frac{(0.2)^5}{5} + \frac{(0.2)^9}{9} - \frac{(0.2)^{13}}{13} + \cdots$$

(f) How many terms are needed to get the integral in **(e)** to within 10^{-6}? Why?

For an alternating series with sum S, $|S - S_n| < |a_{n+1}|$

here $\frac{(0.2)^9}{9} \approx 5.7 \times 10^{-8}$ so S_2 is within 10^{-6} of S

(g) Approximate to with 10^{-6}. Only this part might need a calculator:

$$\int_0^{0.2} \frac{1}{1+x^4}\,dx \approx 0.2 - \frac{(0.2)^5}{5} = 0.199936000$$

(versus more digits via Matlab's quad: 0.1999360568)