GUIDE TO DDEC: STABILITY OF LINEAR, PERIODIC DDES USING THE `DDEC` SUITE

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1. Introduction

The `ddec` suite of programs, written for use in the interactive environment of Matlab, is designed primarily to address the following problem of the stability of systems of linear delay differential equations (DDEs) with periodic coefficients:

Problem. For each vector ϵ of parameter values in some predetermined region of a parameter space $\mathbb{R}^d$, determine the asymptotic stability (i.e. does $x(t) \to 0$ as $t \to \infty$?) of the linear DDE with L integer delays:

$$\dot{x}(t) = A(t,\epsilon)x(t) + \sum_{j=1}^{L} B_j(t,\epsilon)x(t - j\tau). \tag{1}$$

Here $x(t) \in \mathbb{R}^n$. We assume that the $n \times n$ coefficient matrices $A(t,\epsilon), B_j(t,\epsilon)$ are piecewise smooth T–periodic functions of t. We also assume they are analytic functions of the parameter vector $\epsilon \in \mathbb{R}^d$. Finally, we assume that the *coefficient period* T is an integer multiple of the *fundamental delay* τ:

$$T = L\tau. \tag{2}$$

The variety of DDE problems which take the above form is addressed in the examples in this tutorial. The theory (of collocation at the Chebyshev extreme points) which is behind the `ddec` suite is given in [1].

In the future this tutorial will contain discussion of the examples.

To **INSTALL THE SUITE** go to `www.math.uaf.edu/~bueler/DDEcharts.htm` and follow the instructions.

For **DETAILS** of the following tools or examples, type "`help ddecNAME`" or "`help exNAME`" at the command line.

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2. M-FILES IN THE `DDEC` SUITE

Tools.

`ddecchart` Plots a stability chart given an array of spectral radii. *Used by* `ddespect`.

`ddeccheb` Produces Chebyshev extreme points, for collocation, scaled to the interval $[0, \tau]$, and the corresponding spectral differentiation matrix D [5]. *Used by* `ddecivp, ddespect` *and* `ddecU`.

`ddecestm` Estimates the number of collocation points (Chebyshev extreme points) needed to approximate the solution to within a tolerance specified using `ddecset`. *Used by* `ddecivp, ddespect` *and* `ddecU`.

`ddeceval` Evaluates a given function (for instance an initial (history) function) at the collocation points (Chebyshev extreme points). *Used by* `ddecivp`.

`ddecget` Gets values from options structure created by `ddecset`.

`ddecinfo` Graphically displays coefficients and other information in a given descriptor function.

`ddecivp` Solves an initial value problem. Needs DDE descriptor function, a particular parameter value, and a vector of values of the initial (history) function.

`ddecplotx` Plots solutions to initial value problems. *Used by* `ddecivp`.

`ddecspect` Produces an array of spectral radii and plots a stability chart. Needs DDE descriptor function and a range of parameter values.

`ddecset` Sets options for `ddecivp, ddespect` and `ddecU`. See the examples.

`ddecU` Produces a finite matrix approximation of the solution (monodromy) matrix U. Needs DDE descriptor function and a particular parameter value. To extract eigenvalues, eigenmodes and pseudospectrum from U use `eigtool` below [6]. *Used by* `ddecivp` *and* `ddecspect`.

Format of a DDE descriptor function.

`M = coeffs(j,t,eps)` This is a user written `m`–file which describes the coefficients of a particular system of DDEs and returns basic information about the system. `coeffs` accepts as input an integer j, a real number t in the range $[0, T]$ where $T(= L\tau)$ is the period of the coefficients, and a $1 \times d$ row vector of parameters `eps`$=\epsilon$.

For $j < 0$ returns a vector of DDE information `M=[L n tau d]`$=[L\, n\, \tau\, d]$ where L is the number of integer delays, n is the dimension of the system, τ is the fundamental delay, and d is the dimension of the parameter space.

For $j = 0$ returns the $n \times n$ matrix `M`$=A(t, \epsilon)$.

For $j = 1, \ldots, L$ returns the $n \times n$ matrix `M`$=B_j(t, \epsilon)$.

See the examples which follow.

Example descriptor functions.

Start with "⋆" examples.

`excontrol` Second order DDE from a control example [REF?]. Shows use of customized stability chart to find most stable point in parameter plane.

`exmathieu`⋆ Delayed Mathieu equation [3], a second order DDE with nonconstant coefficients.

`exmil` DDE with nonconstant coefficients arising in a milling example.

`exscal`⋆ Scalar DDE with constant coefficients and one delay.

`exscalonep` Scalar DDE with constant coefficients and one delay with only one parameter. Shows use of $d = 1$ parameter in stability chart.

`exturn` DDE with constant coefficients arising in a turning example.

Important related MATLAB tools not in the `ddec` suite.

`dde23` Built-in to MATLAB starting with version 6.5. Solves general initial value problems (IVPs) for DDEs. Superior to `ddecivp` for most nonsmooth IVPs and all nonlinear IVPs. For tutorial see MATLAB documentation (versions 6.5 and later). See [4] for theory of how `dde23` works.

`DDE-BIFTOOL v 2.0` A collection of MATLAB routines for numerical bifurcation analysis of systems of delay differential equations with several constant and state-dependent delays [2].

`eigtool` Given a matrix (for instance, U produced by `ddecU` above), computes eigenvalue and pseudospectrum [6].

References

[1] E. Bueler, *Chebyshev collocation for linear, periodic ordinary and delay differential equations: a posteriori estimates*, 2004. arXiv:math.NA/0409464.

[2] K. Engelborghs, T. Luzyanina, and G. Samaey, *DDE-BIFTOOL v. 2.00: a Matlab package for bifurcation analysis of delay differential equations.* Report TW 330, Dept. of Computer Science, K. U. Leuven, Leuven, Belgium. MATLAB codes available by email to the authors at `koen.engelborghs@cs.kuleuven.ac.be.`, 2001.

[3] T. Insperger and G. Stépán, *Stability chart for the delayed Mathieu equation*, R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci., 458 (2002), pp. 1989–1998.

[4] L. F. Shampine and S. Thompson, *Solving DDEs in MATLAB*, Appl. Numer. Math., 37 (2001), pp. 441–458.

[5] L. N. Trefethen, *Spectral Methods in MATLAB*, SIAM Press, Philadelphia, 2000.

[6] T. G. Wright, M. Embree, and L. N. Trefethen, *EigTool: A MATLAB GUI for computing eigenvalues, pseudospectra and related quantities for matrices.* Available at `www.comlab.ox.ac.uk/pseudospectra/eigtool.`, 2002.

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