What to know about matrix norms: Complete List!

- matrix norms have all vector norm properties:
  $\|A\| = 0 \iff A = 0$, $\|A + B\| \le \|A\| + \|B\|$, $\|\alpha A\| = |\alpha|\|A\|$
- only four norms in widespread use: $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_\infty$, and $\|\cdot\|_{\text{Frob}}$
- three have computable formulas (1, ∞, Frob)
- three are induced from vector norms (1, 2, ∞)
- all four have $\|AB\| \le \|A\|\|B\|$ (but—weirdly—for different reasons)
- always $\rho(A) \le \|A\|$ for any norm, but learn to expect $\rho(A) < \|A\|$
- iteration $v, Av, A^2v, A^3v, \dots$ converges if and only if $\rho(A) < 1$
- thus: if $\|A\| < 1$ then convergence ... *but not conversely*
- $\|\cdot\|_2$ is best for hermitian A: if $A^* = A$ then $\rho(A) = \|A\|_2$
- geometric picture clearest for $\|\cdot\|_2$: image under A of unit ball is ellipsoid with $\|A\|_2$ the length of the semimajor axis
- if A is square: $\text{cond}(A) = \|A\|_2\|A^{-1}\|_2$