Take-home Final Exam

Due Thursday 9 May, 2013, at NOON in my office box

150 points total

As stated on the syllabus, this Final is 25% of your course grade.

Rules. You may not talk or communicate about this exam with any person other than Ed Bueler. You may use any reference book or reference article, print or electronic, as long as it is clearly cited, but you may not search out complete solutions to these particular problems. References to the textbook should be added as needed for clarity.

Exercise 3.1 in Lecture 3. (*10 pts*)

Exercise 6.5 in Lecture 6. (*10 pts*)

Exercise 9.2 in Lecture 9. (*10 pts*)

Exercise 21.1 in Lecture 21. (*10 pts*)

Exercise 22.1 in Lecture 22. (*10 pts*)

Exercise 23.2 in Lecture 23. (*15 pts*)

Exercise 23.3 in Lecture 23. (*10 pts*)

Exercise 24.1 in Lecture 24. (*10 pts*)

Exercise 26.1 in Lecture 26. (*15 pts*)

F1. (*15 pts*) How closely can you fit x^{-1} to a linear combination of e^x, $\cos(x)$, $\ln(x)$ on $[1, 2]$? On $[1, 20]$? Write a MATLAB program that answers these questions. It will use discretizations of the appropriate closed interval. It sets up and solves a discrete least squares problem for each closed interval. Add a sentence or two about your choice of the least squares solution method, explaining the reasoning for your choice of method. Write down your estimate of the answers (i.e. the coefficients of the linear combination). Produce good plots of the optimal approximations; include the function x^{-1} in your plots.

F2. Consider the following 4×4 matrix A:

$$A = \begin{bmatrix} 2 & 3 & 5 & 7 \\ -1 & 2 & 3 & 5 \\ -1 & -1 & 2 & 3 \\ -1 & -1 & -1 & 2 \end{bmatrix}$$

(a) (*5 pts*) Show, using your preferred by-hand method, that A has exact rank 4.

(b) (*10 pts*) Find the rank 2 matrix B which is closest to A in 2-norm. (*You should use* MATLAB *but make it clear what you are doing and why. Write it like it is a textbook example.*)

F3. **(a)** (*10 pts*) Reproduce Figure 12.1 using MATLAB's `roots` command.

(b) (*10 pts*) In your own words, write a paragraph or two—say, at least six good sentences but at most half a page—on how `roots` works and to what degree it succeeds in getting accurate estimates of the roots of the polynomial from the input list of the coefficients.

In particular, consider this extract from the MATLAB technical documentation:[1]

> *The algorithm simply involves computing the eigenvalues of the companion matrix:*
>
> ```
> A = diag(ones(n-1,1),-1);
> A(1,:) = -c(2:n+1)./c(1);
> eig(A)
> ```
>
> *It is possible to prove that the results produced are the exact eigenvalues of a matrix within roundoff error of the companion matrix* `A`*, but this does not mean that they are the exact roots of a polynomial with coefficients within roundoff error of those in* `c`*.*

Your answer should address these sub-questions: In terms of the Lectures in the textbook Trefethen & Bau, what algorithms do you think `eig` uses? What theorem in the textbook comes closest to showing "that the results produced are the exact eigenvalues of a matrix within roundoff error of the companion matrix `A`"? How would you want to generalize the theorem in the textbook to handle the current problem? How does the conditioning of the problem affect the answer?

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[1] See `http://www.mathworks.com/help/matlab/ref/roots.html`.