Assignment # 9

Due Monday 22 April, 2013

Please read Lectures 14, 15, 16, 17, 20, 21, 22, and 23 in the textbook *Numerical Linear Algebra* by Trefethen and Bau. Do these exercises:

Exercise 15.1abcd in Lecture 15.

Exercise 16.2 in Lecture 16.

Exercise 17.2 in Lecture 17. (*Hint*. It is fine to ignore (17.5). That is, the question could be stated "What does (17.4) imply about the error $\|\tilde{x} - x\|$?")

Exercise 17.3a in Lecture 17.

Exercise 20.3a in Lecture 20.

Exercise 20.4 in Lecture 20.

P18. (*Although this was sketched in class and is done implicitly in Lecture 17, it is still a great exercise.*)

(a) Consider the inner product $f(x, y) = x^*y$ for $x, y \in \mathbb{R}^n$ as a problem $f : \mathbb{R}^{2n} \to \mathbb{R}^1$. Write a more-or-less formal proof that the obvious algorithm

$$\tilde{f}(x,y) = \left[\dots \left[\left[[\mathrm{fl}(x_1) \otimes \mathrm{fl}(y_1)] \oplus \mathrm{fl}(x_2) \otimes \mathrm{fl}(y_2) \right] \oplus \mathrm{fl}(x_3) \otimes \mathrm{fl}(y_3) \right] \oplus \cdots \oplus \mathrm{fl}(x_{n-1}) \otimes \mathrm{fl}(y_{n-1}) \right] \oplus \mathrm{fl}(x_n) \otimes \mathrm{fl}(y_n)$$

is backward stable on a machine satisfying (13.5) and (13.7).

(b) Does the order of operations implied by the square parentheses in part **(a)** matter in precisely specifying the algorithm? If so, do you think all of the other orders determine backward stable algorithms, or not? Explain.

(c) Suppose we *fix* $y \in \mathbb{R}^n$ and we consider the same problem as $f(x) = x^*y$ and the same algorithm as $\tilde{f}(x)$. Now $f, \tilde{f} : \mathbb{R}^n \to \mathbb{R}^1$. Modify your proof in part **(a)**, *if needed*, so as to show $\tilde{f}(x)$ is a backward stable algorithm for $f(x)$. (*That is, put the "blame" for all errors on x so that $\tilde{f}(x) = f(\tilde{x})$ for $\tilde{x}$ which is close to, but not equal to, x.*)

(d) Now use the result of part **(c)** to show that for data $A \in \mathbb{R}^{m \times n}$ and *fixed* $y \in \mathbb{R}^n$, the problem of matrix-vector multiplication, i.e. $f(A) = Ay$, has a backward stable algorithm $\tilde{f}(A)$. (*Hint*. The algorithm implements the matrix-vector product by computing inner products only.)