Assignment #2

Due Wednesday 6 February, 2013 at the start of class

Please read Lectures 2, 3, and 4 in the textbook *Numerical Linear Algebra* by Trefethen and Bau. Then do these exercises:

P4. In your undergraduate linear algebra class, or elsewhere, you learned a method for computing determinants called "expansion by minors." Compute this determinant by hand to demonstrate that you know this algorithm:

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\right).$$

Check your work in MATLAB. Now, for an arbitrary $A \in \mathbb{C}^{m\times m}$, count the exact number of multiplication operations needed to compute $\det(A)$ by this method. (*Hint*: How much more work is the $m \times m$ case than the $(m-1) \times (m-1)$ case?)

P5. Consider Trefethen & Bau's Theorem 1.3, which I have called the "fundamental theorem of linear algebra." Retrieve or appropriate an undergraduate text on linear algebra. Find the proof of this theorem in this undergrad text. This proof may be spread across many pages because there are many equivalences. Now carefully write out the proof of two of the equivalences in Theorem 1.3. (*Hint*: Specifically, write two proofs that look like this: "(a) $\iff$ (b): $A \in \mathbb{C}^{m\times m}$ has an inverse A^{-1} if and only if $rank(A) = m$. *Proof.* …")

P6. Write a MATLAB program which draws the unit balls shown on page 18 of Trefethen & Bau. That is, draw clean pictures of the unit balls of $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_\infty$, and $\|\cdot\|_p$. As suggested on page 18 for asthetic reasons, use $p = 4$ for the last one. Don't bother drawing the weighted unit ball (3.4) unless you are curious.

Exercise 1.4 in Lecture 1.

Exercise 2.1 in Lecture 2.

Exercise 2.3 in Lecture 2.

Exercise 2.4 in Lecture 2.