Assignment #5

Due Monday, 8 October 2018, at the start of class

Please read sections 12.1, 12.2 of the textbook, ignoring the Lemmas for now. Please read the online slides *Steepest descent is not great* at

bueler.github.io/M661F18/sdslides.pdf

Problem P9. Let $c \in \mathbb{R}^n$ and suppose $Q \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix. Consider this quadratic function on $x \in \mathbb{R}^n$:

$$f(x) = \frac{1}{2} x^\top Q x - c^\top x. \tag{1}$$

(a) Show that $\nabla f(x) = Qx - c$.
(b) Show that f is strictly convex. (*Hint. You may use facts stated in section 2.3.*)
(c) Suppose p is a descent direction at x, so that $p^\top \nabla f(x) < 0$. Prove that the exact solution of the line search problem $\min_{\alpha > 0} f(x + \alpha p)$ is

$$\alpha = \frac{-p^\top \nabla f(x)}{p^\top Q p}.$$

(*Hint. Define $g(\alpha) = f(x + \alpha p)$, expand it, and compute $g'(\alpha)$. Do mention why is it important that p is a descent direction.*)

Problem P10. In the slides I show a MATLAB implementation of steepest descent using back-tracking. If we restrict the objective function $f(x)$ to only being quadratic then we can use the result in **P9** to choose the step size.

(a) Implement steepest descent with optimal step size for quadratic functions (1):

```
function z = sdquad(x0,Q,c,tol)
```

As before, stop when $\|\nabla f(x_k)\| <$ `tol`.
(*Hint. Only a small modifications needed. Replace evaluations of f and ∇f by formulas for the quadratic case. Replace back-tracking by the result from* **P9**.)
(b) Use `sdquad()` to reproduce the result of Example 12.1 on pages 404–405 of the textbook. Specifically, you should get $k = 216$ iterations using `tol` $= 10^{-8}$.
(c) Now change Q to

$$Q = \begin{pmatrix} 2.3 & 0.19 & -0.89 \\ 0.19 & 1.84 & 0.32 \\ -0.89 & 0.32 & 1.86 \end{pmatrix}$$

but keep the same c, x_0, and `tol` as in part (b). What is x^*? How many iterations does `sdquad()` need? Why is this problem easier than part (b)? (*Hint. What does* `eig(Q)` *tell you?*)