Assignment #4

Due Friday, 5 October 2018, at the start of class

Please read sections 3.1, 3.2, 3.3, 4.1, 4.2, 4.3 of the textbook.

DO THE FOLLOWING EXERCISES from page 82:

- Exercise 1.1
- Exercise 1.3

DO THE FOLLOWING EXERCISES from pages 84–85:

- Exercise 2.1 (ii)–(iv)   *←— I did part* (i) *in class.*
- Exercise 2.2

DO THE FOLLOWING EXERCISES from pages 98–100:

- Exercise 1.1 (i)–(iv)
- Exercise 1.2

Problem P8.   On pages 90–91 the book describes how to use the QR decomposition to build a null-space matrix for A in a numerically-stable way:

> …let A be an $m \times n$ matrix with full row rank. We perform an orthogonal factorization of $A^\top$:
> $$A^\top = QR.$$
> [Then let] $Q = (Q_1, Q_2)$, where Q_1 consists of the first m columns of Q and Q_2 consists of the last $n - m$ columns. [Then]
> $$Z = Q_2$$

Note that an $m \times n$ matrix with full *row* rank has $m \leq n$, so in the description above $n - m$ is either zero or positive. As the book says, the columns of Z are not just a basis for the null space $\mathcal{N}(A)$, but a nice *orthogonal basis* for $\mathcal{N}(A)$.

Write a MATLAB function[1]

```
function Z = mynull(A)
```

which implements the above strategy. In MATLAB the "orthogonal factorization" step can use the function `qr()`; you do not have to worry how `qr()` works. Your code should be quite short. Note that `size(A)` will tell you the values of m and n. Your code should stop if $m > n$.

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[1]In Python, see functions `qr()` and `null_space()` from `scipy.linalg`. These replacement the MATLAB commands `qr()` and `null()` above.

Test your function `mynull()` it on the matrices

$$A_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 4 & 1 & 0 & 1 & 4 \end{pmatrix}$$

and

$$A_2 = \begin{pmatrix} 0 & 1 & 2 & 0 \end{pmatrix}$$

and

$$A_3 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

Are the columns of Z in the null space of the matrix in each case? (*Show command-line* MATLAB *verifications.*) How does the result of `mynull()` differ from the result of the built-in command `null()` on the above matrices? (*Use* `norm` *to answer this.*) Is `null()` implemented the same way as `mynull()`?