Assignment #3

Due Friday, 21 September 2018, at the start of class

Please read sections 2.4, 2.5, 2.6, and 2.7 of the textbook.

DO THE FOLLOWING EXERCISE from page 58:

- Exercise 4.3

DO THE FOLLOWING EXERCISES from page 66:

- Exercise 6.5
- Exercise 6.6

DO THE FOLLOWING EXERCISES from pages 74–75:

- Exercise 7.1
- Exercise 7.2
- Exercise 7.10 $\longleftarrow$ *A sketch of a curve for each equation will help.*

Problem P7. (*This exercise uses a simple,* and inadequate, *algorithm to illustrate "General Optimization Algorithm II" on page 55 of the textbook. We will do much better, though some of the difficulties seen here will remain. This exercise asks you to analyze the deficiencies of this algorithm, not propose a better one! I am sure you can propose a better one.*)

Consider one dimensional unconstrained optimization problems

$$\min_{x \in \mathbb{R}} f(x)$$

for f which has one continuous derivative. I propose the following algorithm:

> **Algorithm P7A.** Assume functions $f(x)$ and $f'(x)$ are supplied.
> 1. Set $x_0 = 1$.
> 2. For $k = 0, 1, 2, \dots$
>   (i) If $|f'(x_k)| < 10^{-3}$ then stop.
>   (ii) If $f'(x_k) > 0$ then let $p_k = -1$. Otherwise let $p_k = +1$.
>   (iii) Let $\alpha_k = 0.01$. Let $x_{k+1} = x_k + \alpha_k p_k$.

a) Implement Algorithm P7A. In MATLAB it will be a function

```
function z = p7a(f,df)
```

where the inputs are functions f = `f` and f' = `df`. The output `z` is the supposed x-coordinate of the solution, i.e. it is the minimizer. (*The handout "Comparison of programming languages ..." gave an example of a function with a function* `f` *as input. You will use this idea in* `p7a.m`.)

b) Run the algorithm, and state briefly what happens, in the following cases:

(i) $f(x) = x^2 - 3x + 2$

(ii) $f(x) = \cos(x/50)$

(iii) $f(x) = e^{\sin(10x)}$

(iv) $f(x) = \operatorname{sech}(x)$

(v) $f(x) = \operatorname{sech}(x - 1)$

c) In at most six complete sentences, describe what you would say are the main deficiencies of Algorithm P7A. Use the results of part **b)** to illustrate some of your points.

d) Note there are magic numbers chosen inside Algorithm P7A. We can put them under the control of the user by changing the algorithm to:

> **Algorithm P7B.** Assume functions $f(x)$ and $f'(x)$ and numbers $x_0 \in \mathbb{R}$, $\epsilon > 0$, and $\delta > 0$ are supplied.
>
> 1. (*The user has supplied* x_0.)
> 2. For $k = 0, 1, 2, \ldots$
>
>   (i) If $|f'(x_k)| < \epsilon$ then stop.
>
>   (ii) If $f'(x_k) > 0$ then let $p_k = -1$. Otherwise let $p_k = +1$.
>
>   (iii) Let $\alpha_k = \delta$. Let $x_{k+1} = x_k + \alpha_k p_k$.

Though you do not need to, one could implement this as a MATLAB function

```
function z = p7b(f,df,x0,eps,delta)
```

In several complete sentences address whether Algorithm P7B is significantly better than P7A. Specifically, consider what the user needs to know about $f(x)$ in order to use P7B effectively.