$\min c^\top x \qquad$ subject to $Ax = b, \quad x \geq 0 \qquad$ where

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & \frac{1}{2} & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 8 \end{bmatrix}, \quad c = \begin{bmatrix} -4 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 8 \end{bmatrix}$$

$$\mathcal{B} = \{3, 4\}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \underline{Bx_B = b} \implies x_B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}, \quad c_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathcal{N} = \{1, 2\}, \quad N = \begin{bmatrix} 1 & 1 \\ 2 & \frac{1}{2} \end{bmatrix}, \quad x_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad c_N = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

$$\underline{B^\top \lambda = c_B} \implies \lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \implies \underline{s_N = c_N - N^\top \lambda} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

$\boxed{s_N \geq 0\text{? stop with optimum}}$ $\quad s_N \xrightarrow{\text{index of min}} q = \boxed{1} \implies \underline{Bd = A_q} \implies d = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\boxed{d \leq 0\text{? stop, unbounded}}$ $\quad \left\{\frac{(x_B)_i}{d_i}\right\} = \left\{ \overset{i=3}{\frac{5}{1}}, \overset{i=4}{\frac{8}{2}} \right\} \xrightarrow{\text{index of min over } d_i>0} p = \boxed{4}$

$$x = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\mathcal{B} = \{3, 1\}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad \underline{Bx_B = b} \implies x_B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad c_B = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$\mathcal{N} = \{4, 2\}, \quad N = \begin{bmatrix} 0 & 1 \\ 1 & \frac{1}{2} \end{bmatrix}, \quad x_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad c_N = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\underline{B^\top \lambda = c_B} \implies \lambda = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \implies \underline{s_N = c_N - N^\top \lambda} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$\boxed{s_N \geq 0\text{? stop with optimum}}$ $\quad s_N \xrightarrow{\text{index of min}} q = \boxed{2} \implies \underline{Bd = A_q} \implies d = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$

$\boxed{d \leq 0\text{? stop, unbounded}}$ $\quad \left\{\frac{(x_B)_i}{d_i}\right\} = \left\{ \overset{i=3}{\frac{1}{3/4}}, \overset{i=1}{\frac{4}{1/4}} \right\} \xrightarrow{\text{index of min over } d_i>0} p = \boxed{3}$

$x = \begin{bmatrix} 11/3 \\ 4/3 \\ 0 \\ 0 \end{bmatrix}$ is x^*

$\mathcal{B} = \{ 2, 1 \}$, $B = \begin{bmatrix} 1 & 1 \\ 1/2 & 2 \end{bmatrix}$, $\underline{Bx_B = b} \implies x_B = \begin{bmatrix} 4/3 \\ 11/3 \end{bmatrix}$, $c_B = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$

$\mathcal{N} = \{ 4, 3 \}$, $N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $x_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $c_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\underline{B^\top \lambda = c_B} \implies \lambda = \begin{bmatrix} -4/3 \\ -4/3 \end{bmatrix} \implies \underline{s_N = c_N - N^\top \lambda} = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$

$\boxed{s_N \geq 0\,?\ \text{stop with optimum}}$ ✓ $\quad s_N \xrightarrow{\text{index of min}} q = \boxed{} \implies \underline{Bd = A_q} \implies d = \begin{bmatrix} \\ \end{bmatrix}$

$\boxed{d \leq 0\,?\ \text{stop, unbounded}} \quad \left\{ \frac{(x_B)_i}{d_i} \right\} = \{ \qquad \} \xrightarrow{\text{index of min over } d_i > 0} p = \boxed{}$

$\mathcal{B} = \{ \qquad \}$, $B = \begin{bmatrix} \\ \end{bmatrix}$, $\underline{Bx_B = b} \implies x_B = \begin{bmatrix} \\ \end{bmatrix}$, $c_B = \begin{bmatrix} \\ \end{bmatrix}$

$\mathcal{N} = \{ \qquad \}$, $N = \begin{bmatrix} \\ \end{bmatrix}$, $x_N = \begin{bmatrix} \\ \end{bmatrix}$, $c_N = \begin{bmatrix} \\ \end{bmatrix}$

$\underline{B^\top \lambda = c_B} \implies \lambda = \begin{bmatrix} \\ \end{bmatrix} \implies \underline{s_N = c_N - N^\top \lambda} = \begin{bmatrix} \\ \end{bmatrix}$

$\boxed{s_N \geq 0\,?\ \text{stop with optimum}} \quad s_N \xrightarrow{\text{index of min}} q = \boxed{} \implies \underline{Bd = A_q} \implies d = \begin{bmatrix} \\ \end{bmatrix}$

$\boxed{d \leq 0\,?\ \text{stop, unbounded}} \quad \left\{ \frac{(x_B)_i}{d_i} \right\} = \{ \qquad \} \xrightarrow{\text{index of min over } d_i > 0} p = \boxed{}$