Assignment #8

Due Monday, 28 November at the start of class

Please read Chapter 12 in Nocedal & Wright, especially the material on pages 304–323. Do the following Exercises and Problems.

Exercise 12.5. There are two different objective functions $f(x)$ to consider. For each one, rewrite the unconstrained minimization problem as a smooth constrained problem. (*Hint*: Consider the example in (12.7) and (12.8) on page 307. Also note that vector norm $\|\cdot\|_\infty$ is defined by (A.2c) on page 600.)

Exercise 12.7.

Exercise 12.13. Ignore the request to show that the MFCQ is satisfied. Just show that the LICQ is *not* satisfied at $x^* = (0,0)$.

Exercise 12.15.

Exercise 12.16.

Exercise 12.17. (*Hint*: Exploit comment (12.35).)

Exercise 12.19. Ignore part **(d)**. In part **(c)**, do find $\mathcal{F}(x^*)$ but ignore the request to write down $\mathcal{C}(x^*, \lambda^*)$.

Problem P21. (*This problem replaces, and expands upon, Exercise 12.20.*) Let $f(x) = x_1 x_2$ and $c_1(x) = x_1^2 + x_2^2 - 1$. Consider the equality-constrained problem

$$\min_{x\in\mathbb{R}^2} f(x) \qquad \text{subject to} \quad c_1(x) = 0.$$

(a) Illustrate the problem with a sketch. How many solutions x^* are there? (*The solution(s) should become clear just from doing the sketch.*) Indicate gradients $\nabla f(x^*)$ and $\nabla c_1(x^*)$.

(b) State the full KKT system (12.34), in detail. Start by giving the index sets $\mathcal{E}, \mathcal{I}$ so it is clear why some parts of (12.34) are empty.

(c) Do algebra on the KKT system to solve the problem. What is the value of λ^* at the solution(s)?

(d) Solve the problem by substituting $x_1(t) = \cos t$ and $x_2(t) = \sin(t)$ and using mild trigonometric knowledge. Confirm the answer is the same as in part **(c)**.