Guide for Final Exam on Friday, 13 December 2019 from 1pm to 3pm

This in-class Final Exam is (again) *closed book* and *closed notes*. Bring only a writing implement. I encourage you to get together and work through this Guide collaboratively. If you come prepared it will be easy.

The 2 hour Exam will have two parts:

Part I. At the start you will have 30 minutes during which you should write a high-quality, one page proof of each of **two theorems from the list below**. You should start by **stating the theorem**. Consider adding any essential definitions, or perhaps a comment, as long as they are brief. Then your **proof should be well-written in complete sentences, with start and end of the proof clearly shown**. The **target length is just longer than one page**.

The emphasis in is on quality and readability. Your presentation should aim at someone who is in a graduate math course but who has not seen the theorem before. The target length is important; $\frac{1}{2}$ page or 2 pages will lose points. Write in pencil during the exam, so that you can erase. If you make a significant mistake then line it out and rewrite a complete sentence. (A big mess will lose points.) You should write out your complete answer when you are preparing for the exam.

Part II. At 1:30pm you will get the rest of the exam, *but you may continue to work on the Part I theorems until the end*. Use time wisely. Part II will look like a midterm, but the questions cover the whole course (see below). Only topics which have appeared on homework or in lecture, or ones which are closely-related, will be asked. Your study for Part II should include looking at Midterms and their solutions.

Theorems allowed for Part I. Only these are allowed. The name of the theorem, its number in the textbook (if stated as a theorem), and the *page numbers* are listed below.

- Banach fixed-point theorem 7.13, the contraction-mapping principle, *98–99*
- Norms in finite-dimensional vector spaces are equivalent, theorem 8.22, *124–125*
- Existence of a nowhere-differentiable function, *157–158*
- Weierstrauss polynomial approximation theorem 11.3, *164–166*
- Lebesgue measure is countably-additive over measurable sets, theorem 16.18, *280*
- Monotone convergence theorem 18.7, *317*

Review Guide for Part II. You are responsible for, and should be able to prove unless otherwise stated, all propositions (lemmas, theorems, and corollaries) in the following material. Likewise you should know all the definitions, unless otherwise stated. Note that some important ideas/definitions/results are stated in the Exercises.

- **Chapter 1:** Everything except Nested Interval Theorem and Bernoulli's Theorem.
- **Chapter 2:** Everything except Bernstein's Theorem.
- **Chapter 3:** Everything here is important. Know the Cauchy-Schwarz, Young's, Hölder's, and Minkowski's inequalities, and study their proofs as well. Review and think about the definitions and basic facts of the ℓ_p spaces.

- **Chapter 4:** Most things here are important except the definitions of "perfect", "isolated point", "boundary point," and the material on the relative metric.
- **Chapter 5:** Everything here is important.
- **Chapter 6:** This material is de-emphasized. However, know the definition of "connected", Theorem 6.6, and that the Intermediate Value Theorem is a special case.
- **Chapter 7:** Almost everything here is important, especially definitions of "totally-bounded," "complete," and "completion" for metric spaces. Note the connection between totally-boundedness and the existence of convergent subsequences. Review Theorems 7.12 and 7.13.
- **Chapter 8:** Again, essentially everything here is important, especially the definition of compact and its open cover definition. The definition of uniform continuity is important, as are Theorems 8.20 and 8.22 and their corollaries.
- **Chapter 10:** The definitions of pointwise and uniform convergence, and spaces $B(X)$ and $C(X)$ for X a metric space, are all fundamental. Consider examples and counterexamples in these spaces. Know Theorem 10.4 (uniform limit of continuous functions is continuous), Theorem 10.5, and Lemma 10.9 (Weierstrauss M-test). Recall Application 10.13 on a nowhere-differentiable function.
- **Chapter 11:** Know the Weierstrauss polynomial approximation Theorem 11.3, and its associated tools (Lemmas 11.1–11.5), plus Application 11.6. I will remind you of Bernoulli polynomial facts on the exam. Skip the material after page 168.
- **David's notes on the Riemann integral:** These notes are fair game. *Make sure you have a copy!* Know the definitions of partitions, step functions, and the Riemann integral. Know basic facts: continuous functions are Riemann integrable, the integral is linear, and the integral is additive over intervals.
- **Chapter 16:** Know everything except the Lebesgue criterion for Riemann integrability and the Vitali Covering Theorem. Reread the introductory material on pages 263–268. Know the definition and major properties of outer measure m^*. Know the definition of measurable sets and basic properties including countable additivity (Theorem 16.18). Know why the set $\mathcal{M}$ of measurable sets is a σ-algebra, and the definition of Lebesgue measure m. Understand material on the structure of measurable sets, but note that we have skipped "G_δ" and "F_σ" stuff. Look at the Cantor set construction in Chapter 2. Know why N is nonmeasurable (Theorem 16.31).
- **Chapter 17:** Everything in this Chapter is fair game, except for Theorem 17.4 and Corollary 17.5. Highlights include the definition of measurable functions, the categorization on bottom of page 298, the proof of Theorem 17.7, the definition of a simple function, the way ∞ is handled (pages 302–303), that a supremum or limit of measurable functions is measurable, the basic construction 17.14, and the approximation of simple and measurable functions by continuous functions.
- **Chapter 18:** Everything is important except for connections to Riemann integrals. Know the definitions of the Lebesgue integral for simple functions, nonnegative measurable functions, and signed measurable functions. Know the Monotone Convergence Theorem, Fatou's Lemma, and the Dominated Convergence Theorem. Know that a nonnegative function f defines a measure. Note Theorem 18.27 and Cor. 18.28.
- **Chapter 19:** Know the material on L_p spaces on pages 342–351, especially definition and completeness for $1 \leq p \leq \infty$, and the Hölder's and Minkowski's inequalities.