

Exercise 0 in Chapter 0

You can write text to explain something, if needed, but you should write in complete sentences. Here we need a definition.

Definition. A real number x is *irrational* if $x \notin \mathbb{Q}$.

Lemmas, propositions, and theorems do not need to be numbered, but please *do* use the theorem and proof environments as shown here.

Theorem. $\sqrt{2}$ is irrational.

Proof. Suppose $x = \sqrt{2} \in \mathbb{Q}$. Then there exist $m, n \in \mathbb{Z}$ so that $x = m/n$ and m, n have no common factors. But then $2 = m^2/n^2$ so $m^2 = 2n^2$. Since m^2 has a factor of 2, and because 2 is prime, it follows that $2|m$, so $m = 2k$ for some $k \in \mathbb{Z}$. Now $(2k)^2 = 2n^2$ or $2k^2 = n^2$ after simplification. Since n^2 has a factor of 2 it follows that $2|n$, but that means both m and n had a factor of 2. This contradicts the assumption that x was rational (and was represented without common factors). \square