Assignment #7

Due Wednesday, 23 October 2019, at the start of class

The Exercises are from Chapters 8 and 10.

One exercise below is identified with your initials. Please LaTeX this problem and send both the `.tex` and `.pdf` to me at `elbueler@alaska.edu` by the same due date as above. See the course website for a LaTeX template.

DO THE FOLLOWING EXERCISES from the textbook:[1]

CHAPTER 8

- Exercise **A**. (*You may use Theorem 8.20, Theorem 8.22, and Corollary 8.23 as needed, but obviously not Corollaries 8.24, 8.25, 8.26. I found it easiest to prove (a), (b), (c) in the given order. Recall that U is finite-dimensional if there is a finite basis, a linearly-independent set S which spans U. You may use the linear algebra fact that if S is a basis for U then for each $x \in U$ there exist unique coefficients $c_j \in \mathbb{R}$ so that x is a (finite) linear combination of elements of S.*)
  - (a) Show that if $(U, \|\cdot\|)$ is a finite-dimensional normed (real) vector space then there exists a linear homeomorphism $F : U \to \mathbb{R}^n$ for some n.
  - (b) Show that if $(U, \|\cdot\|)$ is a finite-dimensional normed vector space then U is complete.
  - (c) Show that if $(W, \|\cdot\|)$ is a normed vector space and $U \subset W$ is a finite-dimensional vector subspace then U is closed.
  - (d) Show that for any $1 \le p < \infty$, the set $V = \{(x_n) : x_n = 0$ for all but finitely-many $n\}$ is a vector (i.e. linear) subspace of ℓ_p. Then show that it is dense. (*And thus not closed!*)
- Exercise #16 on page 110. $\longleftarrow$ **AM**
- Exercise #23 on page 111. $\longleftarrow$ **DD**
- Exercise #57 on page 116. (*Regarding this definition, see also Exercise #59.*)
- Exercise #77 on page 123. $\longleftarrow$ **WV**
- Exercise #78 on page 123.
- Exercise #83 on page 124.

CHAPTER 10

- Exercise #9 on page 149. (*Regarding the term-by-term question, just consider integration, and ignore differentiation.*)
- Exercise #14 on page 151.

[1]Carothers, *Real Analysis*, Cambridge University Press 2000.