Assignment #6

Due Wednesday, 9 October 2019, at the start of class

The problems below are all from Chapter 7.

One exercise below is identified with your initials. Please LaTeX this problem and send both the `.tex` and `.pdf` to me at `elbueler@alaska.edu` by the same due date as above. See the course website for a LaTeX template.

DO THE FOLLOWING EXERCISES from the textbook:[1]

CHAPTER 7

- Exercise **A**. Suppose $f : [a, b] \to \mathbb{R}$ is continuous and differentiable on (a, b), and satisfies $f(a) < 0, f(b) > 0$. Suppose there are constants m, M so that $0 < m \le f'(x) \le M$ for all $x \in (a, b)$.
  (a) Show that there is a unique solution x^* to $f(x) = 0$ on $[a, b]$.
  (b) Show that if $0 < \lambda < 2/M$ then the function $\varphi(x) = x - \lambda f(x)$ has a unique fixed point on $[a, b]$.
  (c) Also show that if $x_0 \in [a, b]$ then $\varphi^n(x_0) \to x^*$. Then estimate the error $|\varphi^n(x_0) - x^*|$ in terms of the first increment $|\varphi(x_0) - x_0|$.

  (*A fixed-point iteration using $\varphi(x) = x - \lambda f(x)$ is sometimes called a* quasi-Newton method *for solving $f(x) = 0$. Do you see the connection to the Newton method?*)

- Exercise #35 on page 97.
- Exercise #41 on page 102. ⟵ **AM**
- Exercise #42 on page 102.
- Exercise #44 on page 105. ⟵ **DD**
- Exercise #48 on page 105. ⟵ **WV**

[1] Carothers, *Real Analysis*, Cambridge University Press 2000.