

ASSIGNMENT #7

(All Problems Due **Friday 11/2/01.**)

**Section 4.3, # 3.**

**Section 4.3, # 5.**

**Section 4.3, # 6.**

**Section 4.3, # 7.**

**Section 4.3, # 8.**

**Section 4.4, # 10.**

**Additional XI.** Suppose  $f : [a, b] \rightarrow \mathbf{R}$ . We define the *lower envelope*  $g$  of  $f$  to be the function defined by

$$g(y) = \sup_{\delta > 0} \inf_{|x-y| < \delta} f(x),$$

and the *upper envelope*  $h$  by

$$h(y) = \inf_{\delta > 0} \sup_{|x-y| < \delta} f(x).$$

Show  $g(x) \leq f(x) \leq h(x)$  for all  $x$ . Show that  $f$  is continuous at  $x$  if and only if  $g(x) = h(x)$ .

[Compare to 2.51 on page 52.]