Assignment #3

(*All Problems Due* **Monday 10/1/01**.)

Section 2.4, # 21.

Section 2.5, # 24.

Section 2.5, # 31.

Section 4.1, # 1.

Section 3.2, # 5.

Additional V. Read section 2.5 on the topology of the real line. Does proposition 9 follow as an immediate corollary from propositions 7 and 8? (That is, did Royden mistakenly use Lindelöf's proof when he could have just written "...follows from propositions 7 and 8"?)

Additional VI. Let

$$g(x) = \begin{cases} \frac{1}{m}, & x = \frac{n}{m} \text{ in lowest terms} \\ 0, & x \text{ irrational} \end{cases}$$

Show g is Riemann integrable and that $\int_0^1 g(x)\,dx = 0$.

Additional VII. (a) Prove that g (in the previous problem) is not continuous at x if and only if x is rational.

[Thus the set of discontinuities is of (Lebesgue and outer) measure zero. It is in fact a general truth (which you are not required to prove): a function f is Riemann integrable on a finite interval $[a, b]$ if and only if the set of discontinuities of f is of measure zero.]

(b) Prove that the function f defined by $f(x) = 1$ if x rational and $f(x) = 0$ if x irrational is discontinuous at every point of the interval $[0, 1]$.

Additional VIII. (*Replaces 3.2 # 6.*) Prove that given any set $A \subset \mathbf{R}$ and any $\epsilon > 0$, there is an open set O such that $A \subset O$ and $M^*O \leq m^*A + \epsilon$.

[**Note:.** The problems from 2.4 and 2.5 are the last review problems!]