Assignment #2

(*All Problems Due* **Friday 9/21/01**.)

Section 1.6, # 23.

Section 1.6, # 24.

Section 2.1, # 3.

Section 2.4, # 7.

Section 2.4, # 11.

Section 3.2, # 5.

Section 3.2, # 8.

Additional II. Let X be an infinite and uncountable set. (You can use $\mathbf{R}^1$ for X here if you want to think concretely—but it won't help!)

We say $A \subset X$ is *cocountable* if $\tilde{A} = X \setminus A$ is countable.

Let $\mathcal{A} = \{A \subset X : A \text{ is countable}\} \cup \{A \subset X : A \text{ is cocountable}\}$. Show that $\mathcal{A}$ is a σ–algebra.

[Note that finite sets are countable by Royden's (and most people's) definition.]

Additional III. Let $C = \{\{1,2,3\},\{3\},\{5\}\}$ be a collection of subsets of the natural numbers $\mathbf{N}$. Completely describe (hint: list all elements) the smallest algebra that contains C. Also describe the smallest σ–algebra that contains C.

Additional IV. Show that the set $\mathbf{Q}$ of rational real numbers is a Borel set and that $m\mathbf{Q} = 0$.

Note. The problems from sections 1.6, 2.1, and 2.4 are review.