Consequences of the Baire theorem

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Definition

if (X, d) is a metric space and $S \subset X$ then we say S is *nowhere dense* if the closure $\overline{S}$ contains no (positive radius) balls

- equivalently, the interior of the closure $(\overline{S})^\circ$ is empty

Theorem (Baire 1899)

a nonempty complete metric space is not a countable union of nowhere dense sets

- that is, if (X, d) is complete and $X = \bigcup_{n=1}^{\infty} A_n$ then there exists a subset A_n whose closure contains a ball: $B_r(x) \subset \overline{A_n}$ for $x \in X$ and $r > 0$
- the proof of the Baire theorem requires the axiom of choice

TITLE

CONTENT

- IT IS OFTEN GOOD TO USE ITEMS FOR SLIDES