Review Guide for In-Class Midterm Quiz on *Wednesday, 22 March 2017*

The Midterm Quiz on 22 March will cover Chapters 1, 2, 3, and 5 of the textbook,[1] and also sections 4.1 and 4.2 and some content from Appendices (see below). The Quiz is *closed book* and *closed notes*.

On this Review Guide I state the *specific* material that will be covered. Material significantly different from this will not be covered. My goal is to only include topics that have appeared on homework and in lecture. The problems will be of these types: state definitions, state theorems and formulas, explain/justify theorems and formulas, give examples with certain properties, or describe or illustrate/sketch concepts. (I will not ask you to "state definition 2.1" or anything like that which refers to a specific location in the book.)

Strongly recommended: Get together with other students and work through this Review Guide. Be honest with yourself about what you do and don't know. Talk it through and learn!

Please ask questions about Quiz content during lecture on Monday 20 March.

Definitions and Notation. Be able to state and use the definition, and/or use the notation/language correctly:

- acronyms: ODE, PDE, IVP, BVP
- order of a differential equation (= maximum number of derivatives in it)
- one-sided and centered finite difference approximations (page 3–4)
- absolute and relative error (Appendix A.1)
- $O(h^p)$ and other "big-oh" notation (Appendix A.2)
- vector norm (Appendix A.3)
- errors in grid functions (Appendix A.4)
- local truncation error (section 2.5: τ^h is the result of applying the scheme to the exact solution)
- global or numerical error (section 2.6: $E_j = U_j - u(x_j)$ or $E = U - \hat{U}$)
- stable method (definition 2.1 in section 2.7)
- consistent method (section 2.8)
- convergent method (section 2.9)
- Poisson equation, Laplace equation, Laplacian operator (page 60, section 3.1)
- 5-point stencil for Laplacian in 2D (section 3.2)
- eigenvalues and eigenvectors (the correct definition is on Assignment #4: $Av = \lambda v$ where $v \neq 0$)
- spectral radius (Appendix C)
- diagonalizable matrix (Appendix C.2)

[1] R. LeVeque, *Finite Difference Methods ...*, SIAM Press 2007

- matrix exponential (Appendix D.3)
- strictly diagonally-dominant matrix (slides associated to Assignment #5)
- Richardson iteration for linear system $Ax = b$ (same)
- Jacobi and Gauss-Seidel iterations (same, *and* sections 4.1, 4.2)
- $f(u, t)$ is Lipschitz continuous in u (formula (5.15) in section 5.2)
- forward Euler, backward Euler, and trapezoid methods (section 5.3)

Formulas, Theorems, and Lemmas. Understand and remember. Be able to illustrate theorems/lemmas with an example, or give a sketch. Use the formula as appropriate to the situation.

- Taylor series (page 5)
- fundamental theorem of finite difference methods (statement (2.22), section 2.9)
- convergence lemma for iterations $y_{k+1} = My_k + c$ (Assignment # 5 and slides)

Techniques. Understand and remember. Be able to illustrate with an example. Use the technique as appropriate to the situation.

- derive a finite difference approximation by the method of undetermined coefficients (page 7)
- set up a finite difference scheme for a first, second, third, or fourth-order ODE BVP (sections 2.4, 2.14, 2.15, 2.16.1)
- Newton's method (section 2.16.1)
- set up a finite difference scheme for Poisson equation in 2D (sections 3.2, 3.3)
- implement Neumann boundary conditions for ODE BVPs (section 2.12)
- convert a higher-order scalar ODE into a system of first-order ODEs (example given in section 5.1)
- take a step of Forward Euler, Backward Euler, or Trapezoid methods (section 5.3)

Make sure you can do these techniques! Practice a few examples. During the Quiz the emphasis will, of course, be on quickly setting up the technique on paper.