Worksheet: Stable time-steps for 2nd-order ODE IVP solvers

Only two rules:
1. *don't* use the internet
2. *do* talk to each other

METHODS. Consider the following three ODE IVP methods, as they apply to an autonomous ODE system $u' = f(u)$. All have $O(k^2)$ local truncation error, and all are stated in the book:

method	*book reference*	*formula*
TR: trapezoidal	(5.22)	$U^{n+1} = U^n + \frac{k}{2}\left(f(U^n) + f(U^{n+1})\right)$
RK2: explicit midpoint	(5.30)	$U^{n+1} = U^n + kf\left(U^n + \frac{k}{2}f(U^n)\right)$
BDF2	p. 173	$3U^{n+2} = +4U^{n+1} - U^n + 2kf\left(U^{n+2}\right)$

PROBLEMS. Consider the following three linear ODE systems:

S1. $u' = Au$ where $u(t) \in \mathbb{R}^3$ and

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & -5 \end{bmatrix}$$

S2. $u' = Au$ where $u(t) \in \mathbb{R}^2$ and

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

S3. $u' = Au$ where $u(t) \in \mathbb{R}^{25}$ and A is the tridiagonal matrix for the $m = 25$ case (i.e. $h = 1/26$ case) of the method-of-lines-discretized heat equation $u_t = u_{xx}$, which models a diffusion process, in problem **P32** on Assignment # 8.

TASKS.

a) For each of the METHODS, write a MATLAB code, presumably using `meshgrid` and `contour` or `contourf`, to plot the absolute stability region.

b) For each of the PROBLEMS, use MATLAB to compute the eigenvalues λ_p of A.

c) For each pair (METHOD,PROBLEM), either determine that there is no stability restriction on the time step, or determine the maximum absolutely-stable time step k_{stab}. For the latter cases, show a figure which has the relevant z-values (i.e. $z = k_{\text{stab}}\lambda_p$) plotted on top of the stability region.

d) For each of the PROBLEMS, give expert advice: which METHOD is best and why? (*Hints.* Generally, think of various ways in which a given method does or does not preserve "qualities" relevant to the particular problem. Computational cost is a consideration too; assume your computer is weak. See section 7.5 and think about k_{stab} and k_{acc}. See sections 8.3 and 8.4.)